FLUID MECHANICS

University of Oulu. Lecturer: A. Ferriz-Mas Spring term 2022

Part I. Basic equations and concepts in Fluid Mechanics

1. Kinematics of the continuum. Spatial and material descriptions.

1.1. Spatial (eulerian) and material (lagrangian) descriptions of the motion. Material derivative. Velocity and acceleration. Trajectories and streamlines.

1.2. Deformation and vorticity tensor fields. Physical interpretation.

1.3. Reynolds' transport theorem.

2. Forces in Continuum Mechanics.

2.1. Long-range (volume/bulk) forces and contact (surface) forces.

2.2. Stress tensor in a continuum distribution of matter.

3. Fundamental equations in Continuum Mechanics.

3.1. Conservation of mass: continuity equation.

3.2. Momentum balance: equation of motion. Kinetic energy theorem.

3.3. Angular momemtum balance: symmetry of the stress tensor.

3.4. Conservation of energy and first principle of Thermodynamics.

3.5. Constitutive relations.

4. Newtonian fluids.

4.1. Hydrostatics. Some astrophysical applications.

4.2. Viscous stress tensor. General definition of pressure.

4.3. Equation of motion for a newtonian fluid: Navier-Stokes equation. Boundary conditions.

5. Energy equation for a newtonian fluid.

5.1. Energy equation. Viscous dissipation function.

5.2. Second principle of Thermodynamics for a linearly viscous fluid.

5.3. Energy equation in the entropic representation. Concepts of adiabatic, isentropic and homoentropic motions.

5.4. Heat conduction. Entropy sources.

6. Circulation and vorticity.

6.1. Circulation and vorticity. Vortex tubes. Some kinematic results for vorticity.

6.2. The equation of motion in terms of vorticity.

6.3. Theorems of Kelvin and Helmholtz for ideal fluids.

6.4. Crocco's equation and Bernoulli's theorems for ideal fluids.

6.5. Enstrophy.

7. Incompressible viscous flow.

7.1. Basic equations. Boundary conditions.

7.2. Scale analysis of the Navier-Stokes' equation. Dimensionless numbers.

7.3. Some elementary viscous flows with an analytic solution: Couette and Poiseuille flows. Rayleigh's impulsive flow; diffusion of vorticity.

7.4. Navier-Stokes' equation for incompressible flows in terms of vorticity. 2-D results.

8. The hydrodynamic equations in conservation form.

8.1. Momentum equation in conservation form. The momentum flux tensor.

8.2. Energy equation in conservation form. The energy flux vector.

8.3. Derivation of the jump relations across a discontinuity. Tangential discontinuities and shock fronts. Rankine-Hugoniot relations.

Part II. Special topics

9. Sound waves in a homogeneous medium.

9.1. Linearization. Wave equation. Characteristic speed.

9.2. Plane and spherical waves. General solution. Boundary conditions and techniques of solutions. Standing waves.

9.3. Energy and momentum of sound waves.

10. Rotating fluids.

10.1. Equation of motion in a rotating frame. The centrifugal acceleration and the geopotencial. The Coriolis acceleration.

10.2. Scale analysis of the equation of motion; dimensionaless numbers. The geostrophic approximation.

10.3. Taylor-Proudman theorem.

10.4. The geostrophic wind and the thermal-wind equation.

11. The virial theorem and astrophysical applications.

11.1. Derivation of the scalar virial theorem in Hydrodynamics. Interpretation of the various terms.

11.2. Some astrophysical applications of the virial theorem: Stars in hydrostatic equilibrium; restriction on the ratio of specific heats γ. Quasistatic contraction as possible energy source. Kelvin-Helmholtz time scale. Free-fall time scale. Derivation of the relationship between the pulsation period and the mean stellar density for pulsating stars.