

# FLUID MECHANICS

University of Oulu. Lecturer: A. Ferriz-Mas

Spring term 2022

## Part I. Basic equations and concepts in Fluid Mechanics

1. Kinematics of the continuum. Spatial and material descriptions.
  - 1.1. Spatial (eulerian) and material (lagrangian) descriptions of the motion. Material derivative. Velocity and acceleration. Trajectories and streamlines.
  - 1.2. Deformation and vorticity tensor fields. Physical interpretation.
  - 1.3. Reynolds' transport theorem.
2. Forces in Continuum Mechanics.
  - 2.1. Volume (bulk) forces and surface (contact) forces.
  - 2.2. Stress tensor in a continuum distribution of matter.
3. Fundamental equations in Continuum Mechanics.
  - 3.1. Conservation of mass: continuity equation.
  - 3.2. Momentum balance: equation of motion. Kinetic energy theorem.
  - 3.3. Angular momentum balance: symmetry of the stress tensor.
  - 3.4. Conservation of energy and first principle of Thermodynamics.
  - 3.5. Constitutive relations.
4. Newtonian fluids.
  - 4.1. Hydrostatics. Some astrophysical applications.
  - 4.2. Viscous stress tensor.
  - 4.3. Equation of motion of a newtonian fluid. Boundary conditions.
5. Energy equation for a newtonian fluid.
  - 5.1. Energy equation. Viscous dissipation function.
  - 5.2. Second principle of Thermodynamics for a linearly viscous fluid.
  - 5.3. Energy equation in the entropic representation. Concepts of adiabatic, isentropic and homoentropic motions.
  - 5.4. Heat conduction. Entropy sources.
6. Circulation and vorticity.
  - 6.1. Circulation and vorticity. Vortex tubes. Some kinematic results for vorticity.
  - 6.2. Theorems of Kelvin and Helmholtz for ideal fluids.
  - 6.3. Crocco's equation and Bernoulli's theorems.

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## 7. Incompressible viscous flow.

- 7.1. Basic equations. Boundary conditions.
- 7.2. Scale analysis of the Navier-Stokes' equation. Dimensionless numbers.
- 7.3. Some elementary viscous flows with an analytic solution: Couette and Poiseuille flows. Rayleigh's impulsive flow; diffusion of vorticity.
- 7.4. Navier-Stokes' equation for incompressible flows in terms of vorticity. 2-D results.

## 8. The hydrodynamic equations in conservation form.

- 8.1. Momentum equation in conservation form. The momentum flux tensor.
- 8.2. Energy equation in conservation form. The energy flux vector.
- 8.3. Derivation of the jump relations across a discontinuity. Tangential discontinuities and shock fronts. Rankine-Hugoniot relations.

## Part II. Special topics

## 9. Sound waves in a homogeneous medium.

- 9.1. Linearization. Wave equation. Characteristic speed.
- 9.2. Plane and spherical waves. General solution. Boundary conditions and techniques of solutions. Standing waves.
- 9.3. Energy and momentum of sound waves.

## 10. The virial theorem and astrophysical applications.

- 10.1. Derivation of the scalar virial theorem in Hydrodynamics. Interpretation of the various terms.
- 10.2. Some astrophysical applications of the virial theorem: Stars in hydrostatic equilibrium; restriction on the ratio of specific heats  $\gamma$ . Quasistatic contraction as possible energy source. Kelvin-Helmholtz time scale. Free-fall time scale. Derivation of the relationship between the pulsation period and the mean stellar density for pulsating stars.

## 11. Rotating fluids.

- 11.1. Equation of motion in a rotating frame. The centrifugal acceleration and the geopotential. The Coriolis acceleration.
- 11.2. Scale analysis of the equation of motion. The geostrophic approximation.
- 11.3. Taylor-Proudman theorem.
- 11.4. The geostrophic wind and the thermal-wind equation.