Irrationality of infinite products

Ondřej Kolouch

Abstract

The talk deals with the criterium for the infinite product of infinite series of rational numbers to be the irrational number which is a joint work of Jaroslav Hančl and Ondřej Kolouch.

In 1975 Erdős proved that if $\{a_n\}_{n=1}^{\infty}$ is an increasing sequence of positive integers such that

$$\liminf_{n\to\infty} a_n^{\frac{1}{2^n}} = \infty$$

then the number

$$\sum_{n=1}^{\infty} \frac{1}{a_n}$$

is irrational.

We follow this result and prove

Theorem. Let $\{a_n\}_{n=1}^{\infty}$ be an increasing sequence of positive integers with

$$\liminf_{n\to\infty} a_n^{\frac{1}{n!}} = \infty.$$

Then the number

$$\prod_{m=1}^{\infty} \left(1 + \sum_{n=0}^{\infty} \frac{1}{a_{n+m} + n}\right)$$

is irrational.