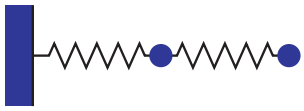
Analyyttinen mekaniikka 763310A
Tentti 13.12.2004

1. Explain shortly the following concepts (a few sentences, possibly a formula for each point)
   (a) Hamilton's principle
   (b) constraint
   (c) Euler angles
   (d) Legendre transformation
   (e) Poisson bracket
   (f) path integral

2. Show that the kinetic energy of a system of particles can be written in the form
$$T = \frac{1}{2}MV^2 + \frac{1}{2}\sum_k m_k v_k'^2, \tag{1}$$
   where $M$ is the total mass, $\boldsymbol{V}$ the velocity of the center of mass, and $\boldsymbol{v}_k'$ the velocity of particle $k$ relative to the center of mass.

3. We study the system shown in the figure, where two particles with identical masses are attached to each other and one of them to a rigid wall by identical springs. The masses can move only in the direction perpendicular to the wall. Construct the Lagrangian, and calculate the frequencies of small oscillations around the equilibrium state.



4. The trajectory of a particle in Earths gravitational field including the Coriolis force is determined by the equation
$$m\ddot{\mathbf{r}} = -mg\hat{\mathbf{z}} - 2m\boldsymbol{\omega} \times \dot{\mathbf{r}}, \tag{2}$$
   where $\boldsymbol{\omega}$ is the angular velocity vector of Earth. We select a coordinate system: $x$ to east, $y$ to north, and $z$ up at the latitude described by the polar angle $\theta$. Show that in throwing a particle towards east with initial velocity $\mathbf{V}$, the Coriolis force, which can be treated as a small perturbation, causes the point where the particle hits to the ground to shift in north-south direction by the distance
$$\Delta y = \frac{4V_x V_z^2 \omega \cos\theta}{g^2}. \tag{3}$$
   Is this distance to north or south?

5. Let us study a particle with kinetic energy $T = \frac{1}{2}mv^2$ and potential energy $V(\boldsymbol{r})$. Let us assume that the generalized coordinates are defined in a time-independent way, $\boldsymbol{r} = \boldsymbol{r}(q_1, q_2, q_3)$. Starting from the definition of the Hamiltonian $H$, show that it is equal to the total energy, $H = T + V$.

Fill in the evaluation form of the course (ask for an english version).