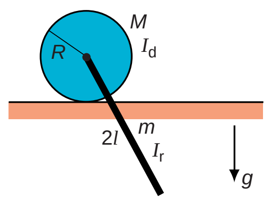
Analytical mechanics 763310A
Examination 8.12.2006

1. We consider a disk (radius R, mass M, moment of intertia $I_d = MR^2/2$ with respect to axis perpendicular to disk going trough the center of the disk) that can roll without slipping on a horizontal line. Attached to the center of mass of the disk with a flexible joint is a rod (length $2l$, mass m, moment of intertia $I_r = ml^2/3$ with respect to an axis perpendicular to the rod going trough the center of mass of the rod). The rod stays in the same plane as the disk (in the plane of the figure). There is uniform gravitational field. Select appropriate coordinates and write the Lagrangian of the system

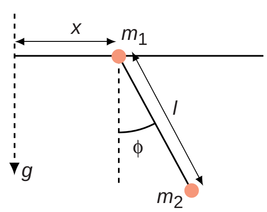


2. State the Hamilton principle. Derive from it the Lagrange equation.

3. The Lagrangian of an oscillator with a gliding hanging point was found to be

$$L = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + \frac{1}{2}m_2 l^2 \dot{\phi}^2 + m_2 l \dot{x}\dot{\phi}\cos\phi + g m_2 l \cos\phi. \qquad (1)$$

Use conservation laws to find all constants of motion. Using them show how to find a formal solution $(x(t), \phi(t))$ for the motion (do not calculate the integrals).



4. The trajectory of a particle in Earths gravitational field including the Coriolis force is determined by the equation

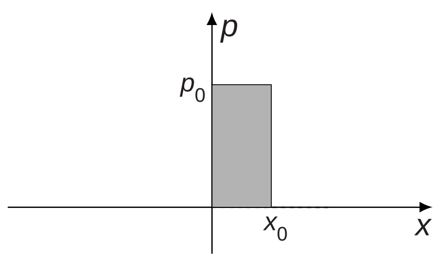
$$m\ddot{\mathbf{r}} = -mg\hat{\mathbf{z}} - 2m\boldsymbol{\omega} \times \dot{\mathbf{r}}, \qquad (2)$$

where $\boldsymbol{\omega}$ is the angular velocity vector of Earth. We select a coordinate system: x to east, y to north, and z up at the latitude described by the polar angle θ. Show that in throwing a particle towards east with initial velocity $\mathbf{V}$, the Coriolis force, which can be treated as a small perturbation, causes the point where the particle hits to the ground to shift in north-south direction by the distance

$$\Delta y = \frac{4V_x V_z^2 \omega \cos\theta}{g^2}. \tag{3}$$

Is this distance to north or south?

5. Let us study a particle that can move freely in one dimension. Write its Lagrangian, Hamiltonian H and the Hamilton equations of motion. Draw the contours of constant H in the phase space. Suppose the initial state of the system at $t = 0$ is described by a probability distribution that has constant non-zero value only in the rectangle shown in the figure. Describe how this distribution will develop in time. State why this satisfies the Liouville theorem.



Fill in the evaluation form of the course (ask for an english version).