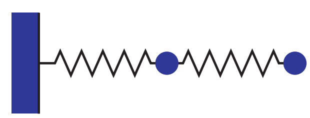
Analytical mechanics 763310A
Examination 18.12.2002

1. Let us study a particle that is constrained to glide without friction on the cylindrical surface parallel to the z axis: $x^2+y^2 = R^2$. A force $\mathbf{F} = -k\mathbf{r}$ directed towards the origin is applied on the particle. Write the Lagrangian. Calculate from it the equations of motion and solve them.

2. State the Hamilton principle. Derive from it the Lagrange equation.

3. Let us study a system where identical masses are coupled to each other and one of them to a rigid wall by identical springs. The masses are constrained to move only in the direction perpendicular to the surface. Write the Lagrangian and calculate the frequencies of small oscillations around the equilibrium configuration.



4. A thin coin rolling on a flat horizontal surface is described by the Lagrangian

$$L = \frac{1}{8}MR^2\left[\dot{\alpha}^2\sin^2\beta + 5\dot{\beta}^2 + 6(\dot{\alpha}\cos\beta + \dot{\gamma})^2\right] - MgR\sin\beta, \quad (1)$$

where M and R are the mass and radius of the coin, α and β the azimuthal and polar angles of its symmetry axis and γ the rotation angle around the symmetry axis. Calculate expressions for the constants of motion.

5. Let us study a particle that can move freely in one dimension. Write its Lagrangian, Hamiltonian H and the Hamilton equations of motion. Draw the contours of constant H in the phase space. Suppose the initial state of the system at $t = 0$ is described by a probability distribution that has constant non-zero value only in the rectangle shown in the figure. Describe how this distribution will develop in time. State why this satisfies the Liouville theorem.