1. **Trapezoidal rule.** Write your a routine that calculates the values of the integral

$$F(x) = \int_{x_0}^{x_1} f(x)\, dx \tag{1}$$

using the trapezoidal rule. Implement the method in a subroutine that takes as its arguments the tabulated values of the integrated function, number of divisions to use and the step length. Routine should return an array of values of the integral function:

```
void trapz(double y[], double h, int n, double yi[])
```

Use your routine to evaluate the integral function

$$F(x) = \int_{0}^{x} \sin^2 y\, dy, \tag{2}$$

where $0 < x < 2\pi$. Choose the discretization length h so that your result matches the known correct value within an accuracy of 10^{-5} at every point.

2. **Numerical integration.** Calculate numerically the integral

$$\int_{0}^{2} x^4 \ln(x + \sqrt{x^2 + 1})\, dx \tag{3}$$

using trapezoidal and Simpsons rules and Romberg integration method. Use Numerical Recipes routines `qtrap`, `qsimp` and `qromb`. Examine how many function evaluations are needed in each of the three methods to reach a desired of accuracy 10^{-6}. Plot the integrand using Gnuplot.

3. **Spline integration.** Using spline integration, calculate the value of the integral function $F(x)$ of

$$f(x) = e^{-x}, \tag{4}$$

for which $F(0) = 0$. Evaluate F at $n = 100$ evenly spaced abscissae on the interval $0 < x < 3$. Plot the integrand and the integral function using Gnuplot.

4. **Gaussian quadratures.** Compute the value of the integral

$$\int_{-1}^{1} \frac{x}{\sqrt{1 - x^2}}\, dx \tag{5}$$

using Gauss–Chebyshev integration.