1. **Householder transformation.** Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 3 & 0 & 4 \\ 3 & -1/3 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 4 & 1 & 0 & 1/4 \end{pmatrix} \tag{1}$$

by hand. Do this by first reducing the matrix to a tridiagonal form using Householder transformation matrices, and then applying a Jacobi rotation matrix on the result.

2. **Differential equations, elementary methods.** Write a program that solves the differential equation

$$\frac{dy(x)}{dx} = f(x,y)$$

$$f(x,y) = \sqrt{x}y - y, \qquad y(0) = 1$$

using given methods. In the following we denote $x_i = x_0 + ih$, $y_i = y(x_i)$.

(a) Euler method, the algorithm is

$$y_{i+1} = y_i + hf(x_i, y_i)$$

(b) Second order Runge–Kutta,

$$k_1 = hf(x_i, y_i), \qquad k_2 = hf(x_i + h/2, y_i + k_1/2)$$

$$y_{i+1} = y_i + k_2$$

3. **Diff. eqs. continued, step-size control** Solve the initial value problem

$$\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right) y = 0,$$

$$y(1) = 0, \qquad y'(1) = 1$$

in the range $1 \leq x \leq 3$. Use Numerical Recipes routine `odeint` and Runge–Kutta with adaptive step-size control.