1. **Spectrum of the hydrogen atom from a differential equation.** In Project Work 3 you need to find the eigenvalues and eigenvectors of Schrödinger equation by discretising the equations and solving them in matrix form. This can also be done by integrating the differential equations.

   The radial part of Schrödinger equation for a hydrogen atom can be written in the form
$$-u''(r) - \left(\frac{2}{r} - \frac{l(l+1)}{r^2}\right) u(r) = Eu(r), \tag{1}$$
   where $u(r)$ are the eigenfunctions and the eigenvalues E are the elctron energies. Setting $l = 1$ and the magnetic quantum number $m = 0$, the wavefunction of the electron is given by
$$\psi(r,\theta) = \mathcal{N} \cos\theta \frac{u(r)}{r}.$$
   The angle θ is the polar angle of spherical coordinates. The probability of finding the electron in a volume element dV is given by the absolute square of the wavefunction, $|\psi(r,\theta)|^2$.

   Find the energies and radial part of the wavefunctions, $R(r) = u(r)/r$, for the lowest three states by integrating Eq. (1), the outline of the method is described in the lecture notes. Also compute the normalization coefficient $\mathcal{N}$ that is obtained from the requirement that the electron is somewhere with a probability of one:
$$\int_V |\psi(r,\theta)|^2 \, dV = \frac{4\pi\mathcal{N}^2}{3} \int_0^\infty u(r)^2 \, dr = 1, \qquad \text{so one has}$$
$$\mathcal{N} = \left(\frac{4\pi}{3} \int_0^\infty u(r)^2 \, dr \right)^{-1/2}.$$
   Finally, compare the eigenvalues you obtained to the known ones,
$$E_n = -\frac{1}{n^2}, \qquad n = 2, 3, \ldots \text{ for } l = 1 \text{ state.}$$