

Cosmology: Homework 2. Solutions.

1: Redshift in Newtonian cosmology. We have that

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} \rightarrow 1 + z = \sqrt{\frac{1 + Hr}{1 - Hr}} = 1 + Hr + \mathcal{O}(H^2 r^2).$$

So if $v = Hr \ll c$, $z = Hr$.

2: Polar coordinates in Euclidean 2-space. Metric is $ds^2 = a^2(dr^2 + r^2 d\phi^2)$. For the spiral, $dr = d\phi = d\eta$, and so the length of the curve is

$$1\text{cm} \int_0^{2\pi} d\eta \sqrt{1 + \eta^2} = 21.256\text{cm}.$$

3: Volume of the 3-sphere. The metric is

$$ds^2 = a^2(d\chi^2 + \sin^2 \chi d\theta^2 + \sin^2 \chi \sin^2 \theta d\phi^2).$$

Then the volume of the spherical shell is

$$\int \sqrt{\det g} d\chi d\theta d\phi = \int_0^{2\pi} d\phi \int_0^\pi d\theta \int_0^\pi d\chi \sin^2 \chi \sin^2 \theta = 2\pi^2 a^3.$$