Cosmology: Homework 8. Solutions.

1: Oldness problem. There are many ways of doing this. Here is one:

The initial energy density is

$$\rho_0 = M_{\rm pl}^4 = \frac{\pi^2}{30} g^* T_0^4.$$

Because this is 0.99 times the critical density, the initial Hubble rate is

$$H_0^2 = \frac{\rho_0}{3M_{\rm pl}^2 0.99} = \frac{M_{\rm pl}^2}{30.99}.$$

First, we note that

$$\Omega \propto a^{-2},$$

so to go from $\Omega = 0.99$ to $\Omega = 0.001$, the scale factor should grow by a factor $a_f/a_0 = \sqrt{990}$. Now integrate the Friedmann equation

$$H_0 t = \int_1^{\sqrt{990}} \frac{dx}{\sqrt{0.01 - 0.99/x^2}} = \frac{1}{0.01}\left(\sqrt{0.99 + 0.01\sqrt{990}^2} - 1\right) = 230.$$

$230 H_0^{-1} = 230 \times 2.97/M_{\rm pl}$ is tiny!

Similarly for redshifting until $T = 2.7$K. We use that $T \propto 1/a$, and so we need

$$x_f = \frac{T_0}{2.7K} = \frac{\left(\frac{30}{\pi^2 10}\right)^{1/4} M_{\rm pl}}{(2.7/11600)\, eV} = 7.75 \times 10^{30}.$$

Then we get

$$H_0 t = \frac{1}{0.01}\left(\sqrt{0.99 + 0.01 x_f^2} - 1\right) = 5.28 \times 10^8,$$

which is $t = 1.5 \times 10^9 M_{\rm pl}^- 1$ and still tiny!

2: Baryon symmetric Universe.

Decoupling takes place when the mean free path is longer than the horizon,

$$H^- 1 < 1/n_N \langle \sigma v \rangle.$$

Now assume radiation domination

$$H^2 = \frac{\pi^2 g^* T^4}{90 M_{\rm pl}^2},$$

and that the nucleons are non-relativistic.

$$n_N = g_N \left(\frac{m_N T}{2\pi}\right)^{3/2} e^{-m_N/T}.$$

Then with $m_N = 938$MeV, $\langle \sigma v \rangle = 135^2 \text{MeV}^2$, $g^* \simeq 10$, $g_N = 4$, we find $T_{\rm dec} \simeq 14.1$MeV.

At that temperature, the baryon-to-photon ratio is

$$\frac{n_N}{n_\gamma} = \frac{g_N \left(\frac{m_N T}{2\pi}\right)^{3/2} e^{-m_N/T}}{2\frac{1.2022 T^3}{\pi^2}} = 7.3 \times 10^{-27}.$$

and it is conserved till this day (assuming no large entropy production in-between. (Actually, the decoupling temperature is around 22MeV, in which case one gets 10^{-19} for the ratio. It's all still much smaller than the observed 6×10^{-11}.)