Cosmology: Homework 9. Solutions.

1: Models of inflation

The slow-roll equations are

$$3H(t)\dot{\phi} = -V', \qquad H^2(t) = \frac{V}{3M_{\rm pl}^2},$$

from which we get (by diving the second by the first

$$H(t) = -\frac{V}{3V'M_{\rm pl}^2}\dot{\phi}.$$

Then take

$$\int_{t_{\mathcal{H}}}^{t_{\rm end}} H(t)dt = -\int_{t_{\mathcal{H}}}^{t_{\rm end}} \frac{V}{3V'M_{\rm pl}^2}\dot{\phi} = \int_{\phi_{\rm end}}^{\phi_{\mathcal{H}}} \frac{V}{3V'M_{\rm pl}^2}d\phi,$$

which is the expression sought.

We have the potential

$$V(\phi) = \frac{\lambda_n}{n}\phi^n.$$

We get

$$\epsilon = \frac{n^2 M_{\rm pl}^2}{2\phi^2} = 1 \rightarrow \frac{\phi_{\rm end}^2}{M_{\rm pl}^2} = n^2/2.$$

We use the equation above to find

$$N = \frac{\phi_{\mathcal{H}}^2 - \phi_{\rm end}^2}{2nM_{\rm pl}^2} \rightarrow \frac{\phi_{\mathcal{H}}^2}{M_{\rm pl}^2} = 2nN + n^2/2.$$

So that

$$\epsilon_{\mathcal{H}} = \frac{n}{4N+n}, \qquad \eta_{\mathcal{H}} = \frac{2(n-1)}{4N+n}$$

We need to satisfy

$$-0.03 = -6\frac{n}{4N+n} + 2\frac{2(n-1)}{4N+n} = -\frac{2n+4)}{4N+n} \qquad n = 1.62.$$

We conclude that $n = 2$ is our best option. Then

$$(2\times 10^{-5})^2) = \frac{\lambda_n}{n150\pi^2 M_{\rm M_{pl}}^{4-n}\epsilon_{\mathcal{H}}}\left(2nN + n^2/2\right)^n,$$

which can just solved, given n, to find λ_n. $n = 2 \rightarrow \lambda_n \simeq 1.5\times 10^{-13} M_{\rm pl}^2$. If we interpret it as a mass $m^2 = \lambda_n$, then the mass of the inflaton is $m \simeq 4\times 10^{-7} M_{\rm pl}$.

We now have the potential

$$V(\phi) = V_0 - \frac{\lambda_n}{n}\phi^n.$$

We get

$$\phi_{\mathcal{H}} = \left(\frac{\lambda_n M_{\rm pl} N}{V_0} + \frac{1}{\sqrt{2}} \frac{\lambda_n M_{\rm pl}}{V_0} \left(\frac{\sqrt{2} V_0}{M_{\rm pl} \lambda_n} \right)^{1/(n-1)} \right)^{1/(2-n)}$$

We introduce

$$x = \frac{\lambda_n M_{\rm pl}^n}{V_0}, \qquad \tilde{\phi} = \phi_{\mathcal{H}} / M_{\rm pl},$$

to get

$$(2 \times 10^{-5})^2 = \frac{V_0 \tilde{\phi}^{2-2n}}{75 \pi^2 M_{\rm pl} x^2}, \qquad -0.03 = -3x^2 \tilde{\phi}^{2n-2} + 2x(1-n) \tilde{\phi}^{n-2}.$$

This can be solved. Note that all of the parameters V_0, λ_n, n enter in both constraints.