1. **Redshift in Newtonian cosmology.** Continuing from problem 1.2, consider the case of critical density ($\kappa = 0$). Denote the distance of galaxy G from the origin by r_G. Solve $r_G(t)$. An observer at the origin sees the light from the galaxies redshifted due to the Doppler effect. For electromagnetic radiation, when the source is moving away from the observer at speed v, we have:

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \sqrt{\frac{1+v}{1-v}}.$$

   Show that at short distances we get the Hubble law for redshift: $z \equiv (\lambda_{\text{obs}} - \lambda_{\text{em}})/\lambda_{\text{em}} = Hr$, where λ_{em} is the emitted and λ_{obs} is the observed wavelength. What approximations do you have to make?

2. Consider polar coordinates in Euclidean 2-d space. The metric is $ds^2 = a^2(dr^2 + r2d\phi^2)$, where $a = 1\,\text{cm}$. Calculate the length of the *spiral*: $r = \eta$, $\phi = \eta$, where η varies from $\eta_1 = 0$ to $\eta_2 = 2\pi$. Draw the spiral.

3. **Volume of the 3-sphere.** Show that the volume of a 3-sphere with curvature radius r (i.e. the surface "area" of a 4d ball of radius r) is $2\pi^2 r^3$.