1. **Energy continuity.** Derive the energy continuity equation

$$\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}$$

from the Friedmann equations. How does this agree with the laws of thermodynamics? (The expansion is adiabatic, $dS = 0$). What does this say about energy conservation?

2. **Conformal time.** It is often useful to make a change in the time coordinate from cosmic time t to *conformal time* η, defined so that

$$d\eta = \frac{a_0}{a(t)}dt, \qquad \text{or} \qquad \eta = a_0 \int_0^t \frac{dt'}{a(t')}.$$

Rewrite the Robertson-Walker metric in conformal coordinates $(\eta, r, \theta, \varphi)$ and $(\eta, \chi, \theta, \varphi)$. Rewrite the Friedmann equation in conformal time.

3. **Matter dominated Universe.** Consider the case of a matter dominated Universe, where $p = 0$, $\rho \propto a^{-3}$. Solve the Friedmann equation for a spatially flat Universe,

$$H^2 = \frac{8\pi G}{3}\rho,$$

for $a(t)$, $\rho(t)$, $H(t)$. How does this fit with the other Friedmann equation? Did you need the assumption $\rho \propto a^{-3}$?