1. **Flat Universe with vacuum energy.** Assume that the cosmological parameters are

$$H_0 = 68.7 \text{ km/s/Mpc}, \quad \Omega_m = 0.273, \quad \Omega_\Lambda = 0.727, \quad \Omega_0 = \Omega_m + \Omega_\Lambda = 1$$

with the other energy components zero. Find the age of the Universe. When (in time t and redshift z) was the matter and vacuum density equal ($\Omega_m = \Omega_\Lambda$)? When did the expansion start accelerating ($\ddot{a} > 0$)?

Hint: You will need so substitute $x^{3/2} = b \sinh \phi$ in the integral

$$\int \frac{x^{1/2} dx}{\sqrt{b^2 + x^3}}.$$

2. **Evolution of the Hubble and density parameters.** Assume a Universe with only non-relativistic matter ($\Omega_m = \Omega_0 \neq 0$, the others $= 0$). Show that the Friedmann equation can be rewritten as

$$H(z) = H_0(1+z)(1+\Omega_0 z)^{1/2}.$$

Derive the evolution of the density parameter with redshift

$$\Omega(z) = \Omega_0 \frac{1+z}{1+\Omega_0 z}.$$