

1. **Age of the closed Universe.** Find the age-redshift relation for a closed Universe ($K > 0$) with $\Omega_\Lambda = 0$. What is the present age of the Universe, if $H_0 = 10\text{km/s/Mpc}$ and a) $\Omega_0 = 1.1$, b) $\Omega_0 = 2$. (Hint, use conformal time).

2. **Gamma ray bursts.** Gamma ray bursts (GRB) from space are observed about once a day. In the late 90's the redshift of some of them was measured, showing that they occur at cosmological distances, which means they are among the most powerful events in the universe. The nature of these explosive events is not yet fully understood. The burst GRB990123, called the "mother of all bursts", was at a redshift $z = 1.6$, had a peak apparent luminosity $l_{peak} = 2.7 \times 10^{-9}\text{W/m}^2$, and a *fluence* (total received energy per unit area, i.e., the intensity integrated over time) of $3.5 \times 10^{-7}\text{J/m}^2$.

What was the peak absolute luminosity and total energy of the burst, assuming GRB's radiate isotropically? Compare the total energy to m_\odot .

(It is actually now thought that the radiation is far from isotropic, making the total power of the brightest bursts far less than what this calculation gives.)

To do this calculation, you need to assume some values for the cosmological parameters:

- a) Use the cosmological parameters $\Omega_m = 0.273$, $\Omega_\Lambda = 0.727$. In this case you probably cannot do the relevant integral analytically, so you can estimate its value from the figures in the lecture notes.
- b) Use instead $\Omega_m = 0.273$ and $\Omega_\Lambda = 0$, and do the integration analytically.