1: Matter-radiation equality. The present density of matter is $\rho_{m0} = \rho_c \Omega_m$, and the present density of radiation is $\rho_{r0} = \rho_{\gamma 0} + \rho_{\gamma 0}$, where $\rho_{\gamma 0} = AT_0^4$ is the microwave background ($T_0 = 2.725$K) and $\rho_{\nu 0} = (21/8)AT_{\nu 0}^4$ is the neutrino background (we assume neutrinos are massless). Here $A = \pi^2/15$, and $T_{\nu 0} = (4/11)^{1/3}T_0$. What was the age of the Universe t_{eq} when $\rho_m = \rho_r$? (Note that in these early times, but not today, you can ignore the curvature and vacuum terms in the Friedmann equation.) Give a numerical value (in years) for the cases $\Omega_m = 0.1, 0.3, 1.0$, and $H_0 = 70$km/s/Mpc. What was the temprature (T_{eq}) then ?

2: Thermal distributions in the non-relativistic limit. Derive the following formulas for non-relativistic Maxwell-Boltzmann statistics ($T \ll m$ and $T \ll m - \mu$) from the general formulas given in the lectures

$$\begin{aligned}
n &= g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m-\mu}{T}}, \\
\rho &= n\left(m + \frac{3T}{2}\right), \\
p &= nT \ll \rho, \\
\langle E \rangle &= m + \frac{3T}{2}, \\
n - \bar{n} &= 2g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}} \sinh\frac{\mu}{T}.
\end{aligned}$$

Here, n is the number density of particles, $\bar{n}$ the number density of corresponding antiparticles.