1: Tremaine-Gunn limit. Assume that neutrinos have a mass, large enough that they are non-relativistic today. This neutrino gas would not be homogeneous, but clustered around galaxies. Assume that they dominate the mass of these galaxies (ignore other matter). We know the mass $M(r)$ within a given radius r in a galaxy from the velocity $v(r)$ of stars rotating around it. The mass could be due to a few species of heavy neutrinos or more species of lighter neutrinos. But the available phase space limits the number of neutrinos with velocities below the escape velocity from the galaxy (you don't need to assume a thermal distribution). This gives a lower limit for the mass of neutrinos m_ν. Assume for simplicity that a) all neutrinos have the same mass or b) only ν_τ is massive. Find (a rough estimate for) the minimum m_ν required for neutrinos to dominate the mass of a galaxy. Assume spherical symmetry and that the escape velocity within radius r is the same as at radius r. Give a value for $v(r) =$220km/s at $r =$10kpc

2: Redshift of non-relativistic particles. Consider a distribution of non-relativistic, non-interacting particles in kinetic equilibrium in an expanding Universe. Using that momentum is redshifted according to $p_2 = (a_1/a_2)p_1$, show that the distribution remains in kinetic equilibrium. How does temperature and chemical potential redshift?