

1: Oldness problem. Assume for simplicity that the Universe contains only radiation, with a constant $g_* = 10$. Suppose the Universe began with a Planck energy density, $\rho = M_{\text{pl}}^4$, and that the density parameter was then less than one, and not extremely close to one, use $\Omega = 0.99$. What is the age of the Universe when Ω becomes smaller than 0.001? What is the age of the Universe when the temperature falls below $T = 2.7\text{K}$?

2: Baryon symmetric Universe. Consider a Universe with an equal amount of nucleons and anti-nucleons (we do not bother distinguishing protons and neutrons). The nucleon-anti-nucleon annihilation cross section satisfies $\langle\sigma v\rangle = \sigma_0 = c_1 m_\pi^2$, with $m_\pi = 135\text{MeV}$, and $c_1 \simeq 1$. The nucleon mass is $m_N = 938\text{MeV}$ and $g_N = 4$ (spin 1/2 and including isospin, protons and neutrons). Estimate the decoupling temperature T_d for nucleon annihilation. Give the amount of relic nucleons as a final nucleon to photon number ratio, and compare it to the observed value $n_N/n_\gamma = 6 \times 10^{-10}$.