1: Models of inflation. In the late 90', there was a whole industry of people constructing models of inflation, based on all sorts of (more or less) motivated forms of the inflaton potential. In the slow-roll approximation, given such a potential, one can easily establish for which choices of parameters the model is consistent with observations of the Cosmic Microwave Background.

This Microwave background has a spectrum (think of it as Fourier transforming a map of temperature fluctuations in the entire sky),

$$\mathcal{P}(k) = A\left(\frac{k}{k_0}\right)^{n_k-1},$$

in terms of an amplitude A and a "spectral index" n_k, which we will assume is independent of scale, $n_k = n$. k_0 is the "pivot scale", ie that scale at which $\mathcal{P}(k) = A$, and sets the normalisation. Observations tell us, that roughly

$$A = \left(2\times 10^{-5}\right)^2, \qquad n-1 \simeq -0.03.$$

Given a potential $V(\phi)$, we now need to calculate A and $n-1$.

The temperature fluctuations originate from tiny quantum fluctuations of the inflaton field during inflation. These in turn depend on the potential V, and it turns out that[1]

$$A = \frac{1}{150\pi^2 M_{\rm pl}^4}\frac{V_{\mathcal{H}}}{\epsilon_{\mathcal{H}}}, \qquad n-1 = -6\epsilon_{\mathcal{H}} + 2\eta_{\mathcal{H}},$$

where ϵ and η are the slow-roll parameters. The subscript $\mathcal{H}$ tells us that we should evaluate these quantities when the scale k_0 "left the horizon", which was sometime during inflation. Let's say N efolds before the end of inflation

$$\ln\left(\frac{a_{end}}{a_{\mathcal{H}}}\right) = N = \int_{t_{\mathcal{H}}}^{t_{end}} H(t)dt = \frac{1}{M_{\rm pl}^2}\int_{\phi_{end}}^{\phi_{\mathcal{H}}}\frac{V}{V'}d\phi.$$

• Establish this equation, using the slow-roll approximation.

In order to test ones model of inflation, one therefore has to do the following (this is what you need to do with the potentials given further down):

1. Choose a potential $V(\phi)$.
2. Find the slow-roll parameters $\epsilon(\phi)$, $\eta(\phi)$.
3. End of inflation is when $\epsilon = 1$ (say...). That gives ϕ_{end}.
4. Given N, we can use the equation above to find $\phi_{\mathcal{H}}$.
5. Compute A and $n-1$ and compare to observations.

• Given $N = 60$ (inflation around the GUT scale), check for what values of λ_m and m the following potentials

$$V(\phi) = \frac{\lambda_m}{m}\phi^m, \qquad m > 2,$$

[1] My convention is to only use $M_{\rm pl}$, the "reduced planck mass", defined as the one that enters in the Friedmann equation as $1/(3M_{\rm pl}^2)$.

satisfy observations. What about $m = 2$?

• In "small field inflation" the inflaton rolls not from large values to small, but from small to large. Which of these

$$V(\phi) = V_0 - \frac{\lambda_m}{m}\phi^m,$$

satisfy observational constraints for $N = 60$? How about $N = 20$ (if inflation ended around the electroweak scale, 100GeV)?