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1. Determine the gradients for the functions

   a) $f(x, y, z) = x^2y + y^2z + z^2x$ in location $(1, -1, 1)$ and

   b) $f(x, y, z) = \cos(x + 2y + 3z)$ in location $(\pi/2, \pi, \pi)$.

   Also determine on these points the tangent planes of the equivalue surfaces.

2. Let $r = |\boldsymbol{r}|$ be the length of the position vector $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$.

   a) Calculate ∇r.

   b) Show that $\nabla f(r) = f'(r)\dfrac{\boldsymbol{r}}{r}$, when $f(r)$ only depends on the length r.

3. Calculate also

$$\nabla^2 \frac{1}{r},$$

   when r is the length of the position vector.

4. Calculate the divergences

   a) $\nabla \cdot (2x^2z\boldsymbol{i} - xy^2z\boldsymbol{j} + 3yz^2\boldsymbol{k})$, b) $\nabla \cdot (r^3\boldsymbol{r})$ and c) $\nabla \cdot [r\nabla(1/r^3)]$.

   Here $\boldsymbol{r}$ is the position vector $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $r = |\boldsymbol{r}|$.