Show to the course assistant latest on 21.10.

1. Are the following series convergent:

   a) $\sum_n \frac{n}{2^n}$,

   b) $\sum_n \frac{n!}{e^n}$,

   c) $\sum_n \frac{(2n)!}{(n!)^2}$

2. Expand as Taylor series $\sum_n a_n x^n$ (around point $x = 0$) the functions

   $$\text{a) } a^x, \quad \text{b) } (1+x)^3, \quad \text{c) } \sqrt{1+x}, \quad \text{d) } \int_0^x e^t \mathrm{d}t$$

3. Determine the radius of convergence for the above series

4. Approximate the value of the integral

   $$\int_{-1}^{1} e^{-x^2} \mathrm{d}x$$

   by expanding the integrand as a Taylor series, integrating term by term and taking into account only a) one, b) two, c) three and d) four first non-zero terms. How does the result deviate from the correct answer 1.49364826...?

   (Note: a function of type $f(x^n)$ are often easiest to expand as a Taylor series by first deriving the series for function $f(z)$, and substituting $z = x^n$ in the resulting series.)

5. Calculate

   $$\text{a) } (1+i)(2-2i), \quad \text{b) } \frac{1+i}{1-i} + \frac{1-i}{1+i}, \quad \text{c) } (1+i)^4$$