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1. Write in polar coordinate form

a)  $-2i$       b)  $-1 + i$       c)  $1 + \sqrt{3}i$   
d)  $\sqrt{3} + i$       e)  $-15$       f)  $0$

2. Find all cubic roots of  $u = 2 + 2i$ , in other words, solutions of the equation  $z^3 = u$ .

3. Show that if  $z \in \mathbb{C}$ , where in polar coordinates  $z = re^{i\phi}$ , complex logarithm function can be defined as follows:

$$\ln z = \ln r + i\phi + i2\pi n, \quad n \in \mathbb{Z}.$$

Hint:  $\ln$  is the inverse function of exponent function. Thus, show that  $e^{\ln z} = z$ . Function  $\ln z$  is an example of a multivalued complex function.

4. Study if

a)  $y = \sin x + x^2$  a solution of  $\frac{d^2y}{dx^2} + y = x^2 + 2$ ,

b)  $x = \cos t - 2 \sin t$  a solution of  $x'' + x = 0$ ,

c)  $\theta = 2e^{3t} - e^{2t}$  a solution of  $\frac{d^2\theta}{dt^2} - \theta \frac{d\theta}{dt} + 3\theta = -2e^{3t}$  and

d)  $y = 3 \sin 2x + e^{-x}$  a solution of  $y'' + 4y = 5e^{-x}$ ?

5. Are the differential equations

a)  $(ye^{xy} + 2x)dx + (xe^{xy} - 2)dy = 0$

b)  $xydx + dy = 0$

c)  $(3r - \theta - 1)d\theta + \theta dr = 0$

separable, exact or linear? The equations can belong to more than one category.