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1. On a small island the number of rabbits is described by function $y(t)$. The number of offspring is assumed to be directly proportional to the number of rabbits. Assuming no predators, the number of rabbits is restricted only by the availability of food, and when the number of rabbits reaches a certain limit L, the available food cannot sustain more rabbits. The rabbit population can be described with the equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ky\left(1 - \frac{y}{L}\right),$$

   where k is the number of children per rabbit in a unit of time.

   a) Find the general solution of the equation (is it separable? Use partial fraction integration).

   b) Let us assume that each pair of rabbits has 4 children per year, i.e. $k = 2/\text{year}$, and that the island can sustain 1000 rabbits ($L = 1000$), and that at the moment $t = 0$ there are 10 rabbits. How many rabbits will there be after 3 years? And after 100 years?

2. Consider differential equation

$$ty'' - (t+2)y' + 2y = 0.$$

   a) Show that the functions $y_1(t) = e^t$ and $y_2(t) = t^2 + 2t + 2$ are linearly independent solutions of the equation.

   b) Find a solution fulfilling initial conditions $y(1) = 0$, $y'(1) = 1$.

3. Find the general solutions for the equations

   a) $y'' + 8y' + 16y = 0$

   b) $u'' + 7u' - 4u = 0$

4. Solve the initial value problems

   a) $y'' + 2y' + y = 0$ when $y(0) = 1$ ja $y'(0) = -3$.

   b) $y'' - 2y' - 2y = 0$ when $y(0) = 0$ ja $y'(0) = 3$.