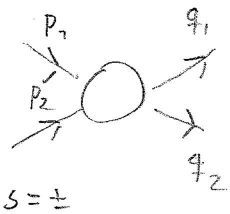
7. Feynman rules

Microscopic physics is encoded in the amplitude $\mathcal{M}$. For a given theory (for example QED, QCD) we can derive a systematic perturbative expansion for calculating it. This is encoded graphically in Feynman-rules: the "language" of particle physics.

Building blocks:

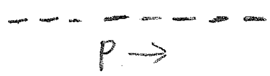
① EXTERNAL LINES



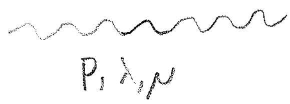
* Fixed 4-momentum
* Spin-½ particles: helicity $s = \pm$
* Spin-1 particles: polarisation λ
* Usually different line types for different particles (in QED, QCD, EW)

contributes to $\mathcal{M}$:

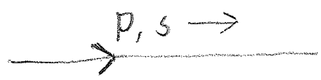
(Real field, i.e. particle = antiparticle)



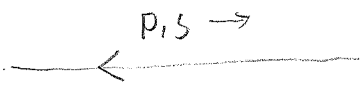
spin-0 $\Rightarrow \delta\mathcal{M} = 1$



spin-1 (photon)
polarisation λ
vector index μ
$\Rightarrow \delta\mathcal{M} = \epsilon^{\mu}_{(\lambda)}(\bar{p})$



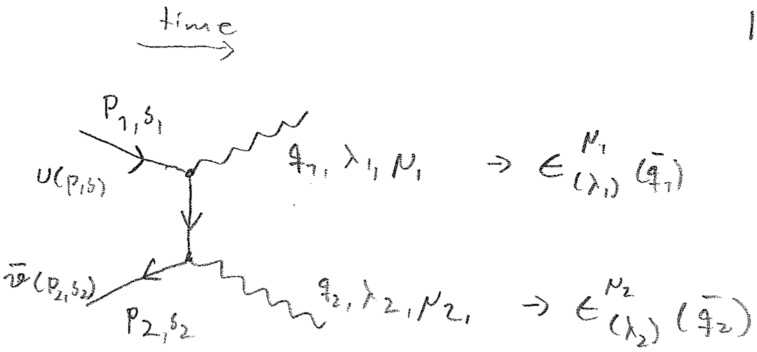
spin-½ (electron)
helicity s
- incoming $\Rightarrow \delta\mathcal{M} = u(\bar{p}, s)$
- outgoing $\Rightarrow \delta\mathcal{M} = \bar{u}(\bar{p}, s)$



spin-½ antiparticle
- incoming $\Rightarrow \delta\mathcal{M} = \bar{v}(\bar{p}, s)$
- outgoing $\Rightarrow \delta\mathcal{M} = v(\bar{p}, s)$

EXAMPLE:

$e^+ + e^- \to \gamma + \gamma$



② VERTICES

(points where lines join together)

* determined by interaction $g\hat{V}$ in $\hat{H} = \hat{H}_0 + g\hat{V}$

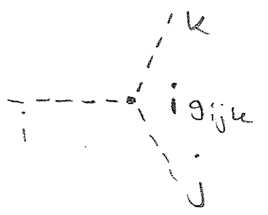
* In Lagrangian formalism

$$-i \int dt\, g \hat{V}_I(t) = i \int dt \int d^3\vec{x}\, \hat{\mathcal{L}}_I$$

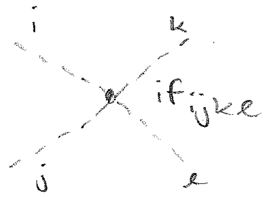
where $\hat{\mathcal{L}}_I$ describes the interaction and is a polynomial in field operators

For example, if

$$\hat{\mathcal{L}}_I = g_{ijk}\, \hat{\phi}_i \hat{\phi}_j \hat{\phi}_k + f_{ijkl}\, \hat{\phi}_i \hat{\phi}_j \hat{\phi}_k \hat{\phi}_l$$



$\Rightarrow \quad \delta M = i g_{ijk}$

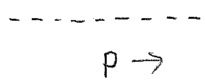


$\Rightarrow \quad \delta M = i f_{ijkl}$

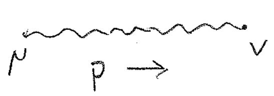
* Coefficients of field operators are coupling constants (g). Each vertex contributes an amount $i \times$ (coupling constant)

3 INNER LINES

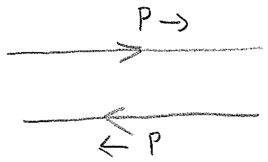
* If the diagram has <u>inner lines</u>, each of these contribute a propagator.
* Inner lines do not have a definite spin or polarization (all possibilities are summed over)



$$\Rightarrow \quad SM = \frac{i}{p^2 - m^2}$$



$$\Rightarrow \quad SM = \frac{i}{p^2}(-g_{\mu\nu})$$



$$\Rightarrow \quad SM = \frac{i}{p^2 - m^2}(\not{p} + m)$$

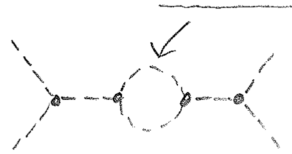
* Terms in parenthesis arise due to completeness relations

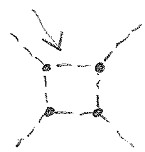
4 4-MOMENTUM CONSERVATION

* Energy and momentum conserved at each vertex

5 INTEGRATION OVER INTERNAL MOMENTA

* Internal momenta are integrated with measure $\int \frac{d^4p}{(2\pi)^4}$
* Because of 4, this is non-trivial only when there are <u>internal loops</u> (example page)





⑥ ANTISYMMETRISATION

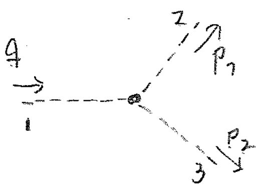
* (-1) (minus-sign) for each closed fermion loop
* (-1) between 2 diagrams which differ <u>only</u> through exchange of 2 external fermion lines

⑦ OVERALL PHASE

* Everything is still multiplied by $(+i)$ (has no effect for $|\mathcal{M}|^2$)

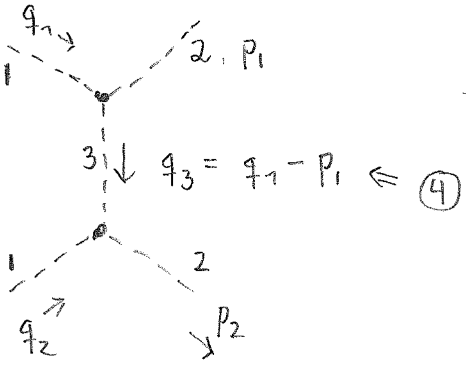
Example : consider $\hat{\mathcal{L}}_I = -g\,\hat{\phi}_1 \hat{\phi}_2 \hat{\phi}_3$

a)



$$\mathcal{M} = \underset{②}{(-ig)} \times \underset{⑦}{(i)} = i$$

b)

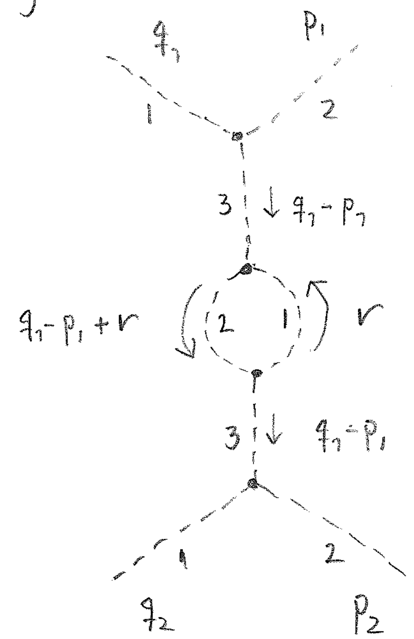


<u>11 → 22 -scattering</u>

$$\mathcal{M} = \underset{②}{(-ig)^2} \times \underset{③}{\frac{i}{(q_1-p_1)^2 - m_3^2}} \times \underset{⑦}{(i)}$$

$$= \frac{g^2}{(q_1-p_1)^2 - m_3^2}$$

c)



One 2nd order contribution to $11 \to 22$

← loop momentum r is not determined by external lines

$$\mathcal{M} = \underset{②}{(-ig)^4} \times \underset{③}{\left[\frac{i}{(q_1-p_1)^2-m^2}\right]^2} \int \underset{⑤}{\frac{d^4 r}{(2\pi)^4}} \underset{③}{\frac{i}{r^2-m^2} \frac{i}{(q_1-p_1+r)^2-m^2}} \times \underset{⑦}{(i)}$$

$$= i g^4 \frac{1}{(q_1-p_1)^2-m^2} \int \frac{d^4 r}{(2\pi)^4} \frac{1}{(r^2-m^2)((q_1-p_1+r)^2-m^2)}$$

8 QED, quantum electrodynamics

* Feynman, Schwinger, Tomonaga, Dyson 1946-51
* Complete description for electromagnetic interactions, which are behind <u>everything</u> from atoms upward!
* Perhaps the <u>most accurately verified theory known</u>!

For example, electron magnetic moment

$$\frac{\mu_e}{\mu_B} = 1.00115965 2187(4) \quad \text{experiment}$$

$$1.00115965 2201(27) \quad \text{theory}$$

Bohr magneton, $\mu_B = e\hbar/2m_e$

* Interaction : simply

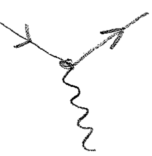
$$\boxed{\hat{\mathcal{L}}_I = e\,\hat{\bar{\psi}}\gamma^\mu\hat{\psi}\,\hat{A}_\mu}$$

Electron - photon interaction

* e = elementary charge, $\frac{e^2}{4\pi} = \alpha = \frac{1}{137.036}$

$\left(\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \text{ in SI}\right)$

* Other charged fermions : $\underline{-e \to q}$

Vertex:  $\underline{ie\gamma^\mu}$

8.1. Leading processes

* First order

- No $e \to e\gamma$ (or $e\gamma \to e$) scattering; (free photon)
however, e can interact with magnetic field

$$\frac{\mu_e}{\mu_B} = 1$$

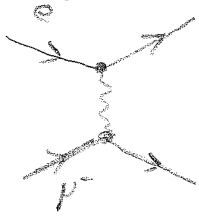
(magnetic field, not propagating)

* Second order

- Elastic

• Electron-muon

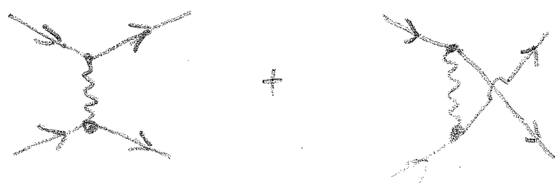
$e^- + \mu^- \to e^- + \mu^-$



(when $m_\mu \to \infty$ this is called "Mott scattering", when $\vartheta_e \to 0$ "Rutherford scattering". In the latter case the sign of charge plays no role ($e + p \to e + p$))

• Electron-electron, "Møller scattering"

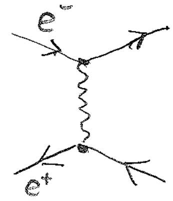
$e^- + e^- \to e^- + e^-$

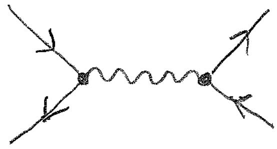
$\mathcal{M} =$ 

outgoing electron can be either one of the incoming ones!

• Electron-positron, "Bhabha scattering"

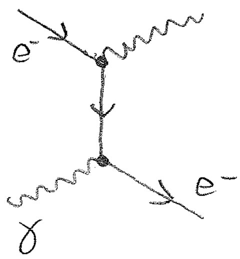
$e^- + e^+ \to e^- + e^+$

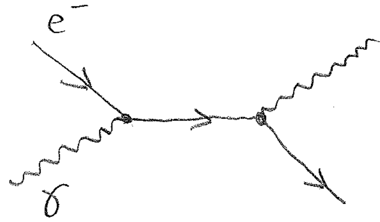
$\mathcal{M} =$ 

$+$ 

• Electron-photon, "Compton scattering"

$e^- + \gamma \to e^- + \gamma$

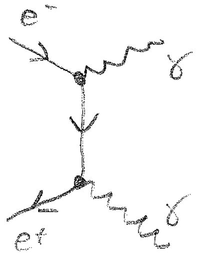
$\mathcal{M} =$ 

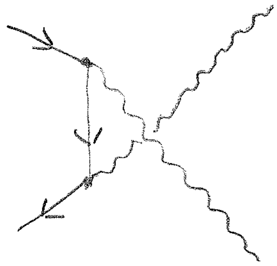
$+$ 

- <u>Inelastic</u>

• Pair annihilation (

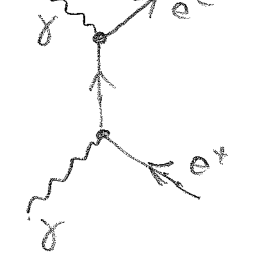
$e^- + e^+ \to \gamma + \gamma$

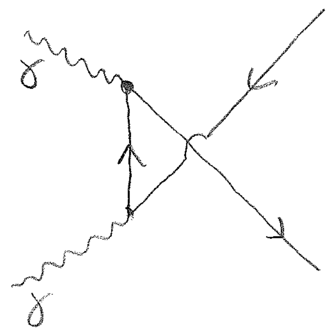
$\mathcal{M} =$ 

$+$ 

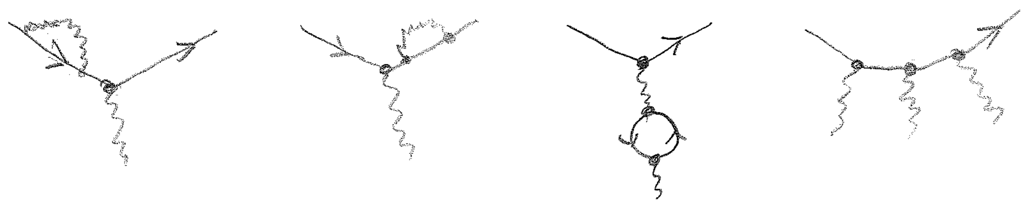
• Pair creation

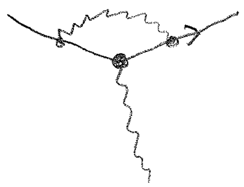
$\gamma + \gamma \to e^+ + e^-$

$\mathcal{M} =$ 

$+$ 

* <u>Third order</u> - lot of processes; important ones for μ_e are





"anomalous magnetic moment"

The size of the higher order corrections can be estimated as follows:

- 2 vertices added $\Rightarrow e^2$
- 1 loop integral $\Rightarrow \frac{1}{(2\pi)^4} \int d\Omega_4 = \frac{1}{(4\pi)^2}$

$$\Rightarrow \frac{1}{4\pi} \frac{e^2}{4\pi} = \frac{\alpha}{4\pi} \simeq \frac{1}{1700}$$

Real answer is $\alpha/2\pi \simeq 0.0011614$ (comp. $\frac{\mu_e}{\mu_B}(\mathrm{exp}) \simeq 1.001159\ldots$)

$\rightarrow$ Corrections small, pert. expansion converges quickly

Mathematically, expansion only asymptotic - does not really converge! Not a problem at low orders