* In ultrarelativistic limit $m \to 0$. Now

$$\mathrm{Tr}[\gamma^\mu \not{q}_1 \gamma^\nu \not{p}_4 \gamma_\mu \not{q}_2 \gamma_\nu \not{p}_3] \qquad \gamma_\mu \not{a}\not{b}\not{c}\gamma^\mu = -2\not{c}\not{b}\not{a}$$

$$= -2\,\mathrm{Tr}[\not{p}_4 \gamma^\nu \not{q}_1 \not{q}_2 \gamma_\nu \not{p}_3] \qquad \gamma_\mu \not{a}\not{b}\gamma^\mu = 4\,a\cdot b$$

$$= -8\, q_1\cdot q_2\, \mathrm{Tr}[\not{p}_4 \not{p}_3] = \qquad \mathrm{Tr}[\not{a}\not{b}] = 4\,a\cdot b$$

$$= -32\, q_1\cdot q_2\; p_4\cdot p_3$$

In the center of mass frame, $\bar{q}_1 = -\bar{q}_2$; $\bar{p}_4 = -\bar{p}_3$;

$E = q_1^0 = q_2^0 = p_3^0 = p_4^0$ (produced particles = initial particles)

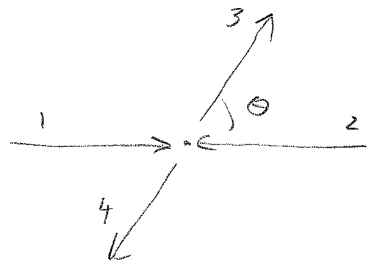
$|\bar{q}_1| = |\bar{p}_3| = E$

$$\Rightarrow \quad q_1\cdot q_2 = E^2 + E^2 = 2E^2 = p_4\cdot p_3$$

We can combine this with result in page 137 (with $m_e = m_\mu = 0$), and

$$q_1\cdot p_4 = E^2(1+\cos\theta) = q_2\cdot p_3$$

$$q_1\cdot p_3 = E^2(1-\cos\theta) = q_2\cdot p_4$$



$$(q_1 - p_3)^2 = \underbrace{q_1^2 + p_3^2}_{0} - 2 q_1\cdot p_3$$

$$(q_1 - p_4)^2 = \overbrace{q_1^2 + p_4^2} - 2 q_1\cdot p_4$$

$$\Rightarrow \langle |M|^2\rangle = \langle |M_1|^2\rangle + \langle |M_2|^2\rangle + \langle M_1 M_2^* + h.c.\rangle$$

$$= \frac{2e^4}{(1-\cos\theta)^2}\left(4 + (1+\cos\theta)^2\right) + \frac{2e^4}{(1+\cos\theta)^2}\left(4 + (1-\cos\theta)^2\right)$$

$$+ \frac{2e^4}{1-\cos^2\theta} \times 8$$

This can be written in terms of Mandelstam variables as

$$\langle |M|^2 \rangle = 2e^4 \left\{ \frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} + \frac{2s^2}{tu} \right\}$$

where in CM-frame

$$s = (q_1+q_2)^2 = 2q_1 \cdot q_2 = 4E^2$$

$$t = (q_1 - p_3)^2 = -2q_1 \cdot p_3 = -2E^2(1-\cos\theta)$$

$$u = (q_1 - p_4)^2 = -2q_1 \cdot p_4 = -2E^2(1+\cos\theta)$$

or, combining all

$$\langle |M|^2 \rangle = \frac{2e^4}{t^2u^2}\Big(s^2 \underbrace{(u^2+t^2+2tu)}_{\underbrace{(t+u)^2}_{s^2 \text{, Exercise 9.1.}}} + u^4 + t^4\Big)$$

$$= \frac{2e^4}{t^2u^2}(s^4+u^4+t^4) = \frac{2e^4}{(q_1\cdot p_3)(q_1\cdot p_4)}\left((q_1\cdot q_2)^2 + (q_1\cdot p_3)^2 + (q_1\cdot p_4)^2\right)$$

Note - these are valid in any frame