10. Weak interactions

* As opposed to QED and QCD, weak interactions are characterized by several "non-conservation" laws: P, CP, flavour (u,d,s ...). These are conserved in QED and QCD; thus, when a process violates these, it must be due to weak interaction

* This makes weak interactions observable! An additional weak interaction which obeys all conservation laws would be unobservable, due to smallness of the interaction.

10.1. Fermi theory - historical perspective

* Already in 1932 Fermi presented a very successful theory for weak processess (much before quarks or Standard Model):

  Consider the weak decay:

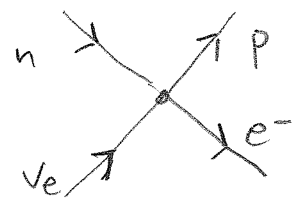
$$n \to p^+ + e^- + \bar{\nu}_e$$

Fermi suggested interaction Lagrangian

$$\hat{\mathcal{L}}_I = -G_F\left[\hat{\bar{n}}\gamma^\mu \hat{p}\, \hat{\bar{\nu}}_e \gamma_\mu \hat{e} + \hat{\bar{p}}\gamma^\mu \hat{n}\, \hat{\bar{e}}\gamma_\mu \hat{\nu}_e\right]$$

Here $\hat{n}, \hat{p}, \hat{\nu}_e, \hat{e}$ are field operators, and

<u>G_F = Fermi coupling</u>



$G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

* G_F fixed by observed lifetime of n

* More modern way to write Fermi theory is to use quarks:

$$\hat{\mathcal{L}}_I = -G_F \left[\hat{\bar{d}} \gamma^\mu \hat{u}\ \hat{\bar{\nu}}_e \gamma_\mu \hat{e} + \text{h.c.} \right]$$

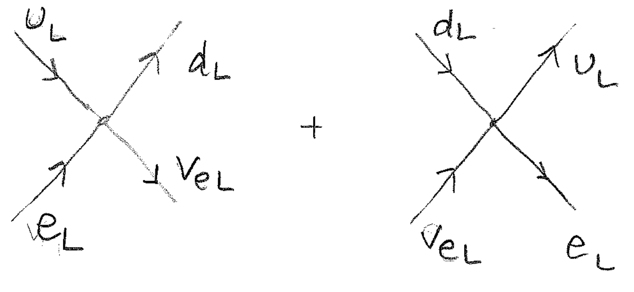
(still not the final theory of weak interactions!)

<u>Parity violation</u>: as discussed in 3.8.1. and page 101, weak interactions violate parity by affecting only <u>left chiral components</u> of fermion fields. Fermi theory needs to be <u>modified</u>:

$$P_L = \tfrac{1}{2}(1-\gamma_5), \quad P_R = \tfrac{1}{2}(1+\gamma_5); \quad \psi_L = P_L \psi, \quad \psi_R = P_R \psi$$

$$\rightarrow \mathcal{L}_I^{V-A} = -2\sqrt{2}\, G_F \left[\bar{d} \gamma^\mu P_L \hat{u} \cdot \bar{\nu}_e \gamma_\mu P_L e + \bar{u} \gamma^\mu P_L d\ \bar{e} \gamma_\mu P_L \nu_e \right]$$

exercise $\rightarrow$ $$= -2\sqrt{2}\, G_F \left[\bar{d}_L \gamma^\mu u_L\ \bar{\nu}_{eL} \gamma_\mu e_L + \bar{u}_L \gamma^\mu d_L\ \bar{e}_L \gamma_\mu \nu_{eL} \right]$$



(This is called Vector-Axial vector (V-A) model, because $\gamma^\mu P_L = \gamma^\mu - \gamma^\mu\gamma^5$ = vector - (axial vector))

* Recall that for massless particles,

  left-handed $\rightarrow$ helicity $h=-1$ (ν_e)

  right-handed $\rightarrow$ helicity $h=+1$ $(\bar{\nu}_e)$

* $\psi = \psi_L + \psi_R$. However, neutrinos always left-handed (antineutrinos right-handed) (if we neglect tiny ν mass)

* In $\mathcal{L}_I^{V-A}$, $\ldots \bar{\nu}_{eL}\gamma_\mu e_L + \ldots \bar{e}_L\gamma_\mu \nu_{eL}$

  ν_e-terms describe $\begin{cases} \text{outgoing } \nu_e, \text{ left} \\ \text{incoming } \bar{\nu}_e, \text{ right} \end{cases}$ $\qquad \begin{cases} \text{incoming } \nu_e, \text{ left} \\ \text{outgoing } \bar{\nu}_e, \text{ right} \end{cases}$

* Generalises to other generations: e.g.

  $\begin{cases} u \rightarrow c \\ d \rightarrow s \end{cases}$ $\qquad \begin{cases} e \rightarrow \mu \\ \nu_e \rightarrow \nu_\mu \end{cases}$

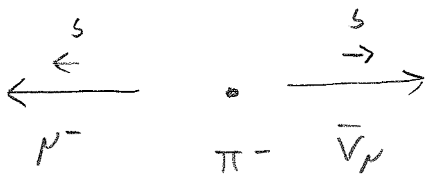
  Using $Q_1 = \begin{pmatrix} u \\ d \end{pmatrix}$; $Q_2 = \begin{pmatrix} c \\ s \end{pmatrix}$; $Q_3 = \begin{pmatrix} t \\ b \end{pmatrix}$ $\begin{matrix} \leftarrow Q = 2/3 \\ \leftarrow Q = -1/3 \end{matrix}$

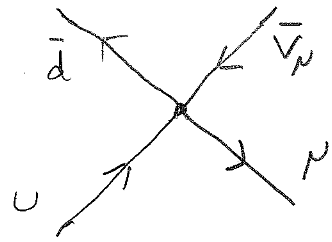
  $L_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$; $L_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$; $L_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ $\begin{matrix} \leftarrow Q = 0 \\ \leftarrow Q = -1 \end{matrix}$

  we can write the V-A lagrangian as

$$\mathcal{L}_I^{V-A} = -2\sqrt{2}\, G_F \sum_{\substack{D_a, D_b \\ \in \{Q_1, Q_2, Q_3, L_1, L_2, L_3\}}} \left[\bar{D}_{aL} \begin{pmatrix} 0 & 0 \\ \gamma^\mu & 0 \end{pmatrix} D_{aL} \times \bar{D}_{bL} \begin{pmatrix} 0 & \gamma_\mu \\ 0 & 0 \end{pmatrix} D_{bL} \right]$$

* For example $\pi^- \to \mu^- + \bar{\nu}_\mu$ (page 73)





* Vertices always switch $\begin{matrix} u \\ d \end{matrix}$ $\begin{matrix} \nu_e \\ e \end{matrix}$

* However, the Lagrangian does not describe flavour violations, e.g.

$K^+ \to \pi^+ + \pi^0$

$s = 1$ $\qquad$ $s = 0$ $\qquad$ strangeness violation

$D^0 \to K^- \pi^+ \pi^0$ charm violation

How this is accomplished? Weak interactions actually mix families (1,2,3). This can be parametrized as (in families 1,2):

$$\left.\begin{matrix} Q_1 = \begin{pmatrix} u \\ d' \end{pmatrix} \\ \\ Q_2 = \begin{pmatrix} c \\ s' \end{pmatrix} \end{matrix}\right\} \text{ where } \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

θ_c = Cabibbo angle $\simeq 13.1^\circ$

* Thus, downstairs-quarks are a <u>mixture</u> of original d, s-quarks

* Why downstairs? Sufficient to do either up or down; convention

* Generalises to mixing of $\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = M \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

  M = Cabibbo-Kobayashi-Maskawa matrix

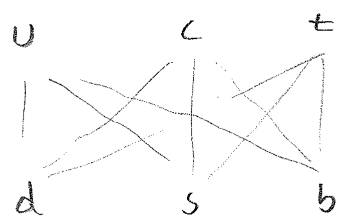
* What is the difference between d and d'?

  - d, s, b mass (eigen)states
  - d', s', b' weak interaction basis

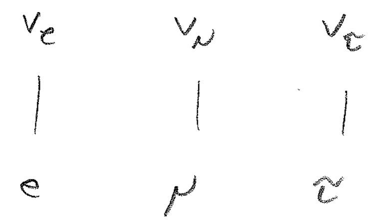
  Thus, if $m_d = m_s = m_b$ we could just use d', s', b' and there would be no mixing

* Because $m_\nu = 0$ (almost), <u>no</u> mixing in the lepton sector

* Thus, weak vertices mix



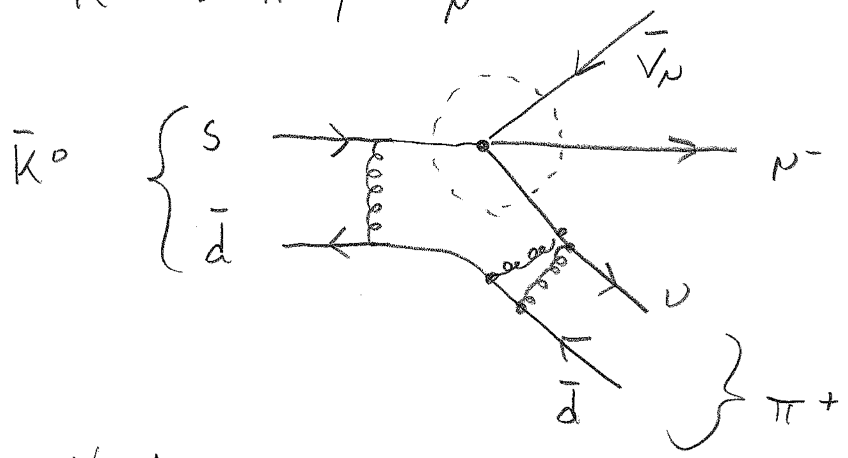
B conserved



L_i conserved
$_{1,2,3}$

Example :

$\bar{K}^0 \to \pi^+ \mu^- \bar{\nu}_\mu$



Vertex comes from

$$\mathcal{L}_I = -2\sqrt{2}\, G_F\, \bar{\mu}_L \gamma^\mu \nu_{\mu L}\; \bar{u}_L \gamma_\mu d'_L$$

outgoing u

← incoming d'_L, contains s!

outgoing μ

outgoing $\bar{\nu}_\mu$

$$\Rightarrow\; -2\sqrt{2}\, G_F \sin\theta_c\, \bar{\mu}_L \gamma^\mu \nu_{\mu L}\; \bar{u}_L \gamma_\mu s_L$$

Thus, the amplitude is

$$\mathcal{M} = -\frac{1}{\sqrt{2}} G_F \sin\theta_c\, \bar{u}(p_\mu)\gamma^\mu(1-\gamma^5) v(p_{\nu_\mu})$$
$$\times\, \bar{u}(p_u)\gamma_\mu(1-\gamma^5) u(p_s)$$

and we can calculate $|\mathcal{M}|^2$ and lifetime of $\bar{K}^0$ (page 109)

10.2. W and Z bosons

* The Fermi theory described in 10.1. works admirably when total energy $\sqrt{s} \ll G_F^{-1/2}$.
(Includes most weak decays)
* However, it fails to describe higher energy processes – it is not even internally consistent!
(renormalizable, chapter 9)

* At best, an effective low-energy description

Solution:

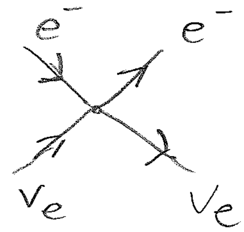
i) let us rewrite $G_F = \dfrac{g_W^2}{4\sqrt{2}\, m_W^2}$, where parameters g_W, m_W are a priori unknown

ii) Denote

$$J^\mu = \sum_{D_L} \bar{D}_L \begin{pmatrix} 0 & 0 \\ \gamma_\mu & 0 \end{pmatrix} D_L \;;\quad J^{\mu +} = \sum_{D_L} \bar{D}_L \begin{pmatrix} 0 & \gamma_\mu \\ 0 & 0 \end{pmatrix} D_L$$

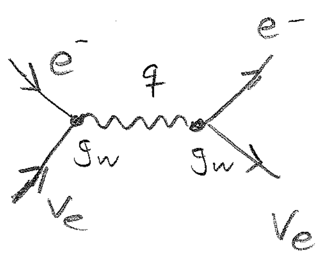
thus, Fermi-model interaction

$$\hat{\mathcal{L}}_I = - \frac{g_W^2}{2 m_W^2} J^\mu g_{\mu\nu} (J^\nu)^+$$



iii) Introduce a "photon" with mass m_W:

$$\hat{\mathcal{L}}_\perp = -i \frac{g_W^2}{2} J^\mu \underbrace{\frac{-i g_{\mu\nu}}{q^2 + m_W^2}} (J^\nu)^\dagger$$



coupling g_W

like a "photon" but with a mass m_W $= W^-$

iv) For low energy processes $q^2 \ll m_W^2$, and we get Fermi theory

At very high energy, $q^2 \gg m_W^2$, and we get (effectively) massless photon-like interaction

$\Rightarrow$ fully renormalisable, healthy theory!

W^- and its antiparticle W^+ were discovered at Cern in 1983, with $m_W = 80.425 \pm 0.038$ GeV.

* Third "intermediate vector boson" Z^0 couples to "neutral currents"

$$J_0^\mu = \sum_D \bar{D} \begin{pmatrix} c_V \gamma^\mu + c_A \gamma^\mu \gamma^5 & 0 \\ 0 & c_V \gamma^\mu + c_A \gamma^\mu \gamma^5 \end{pmatrix} D$$

with certain coefficients c_V, c_A

(this is <u>not</u> within V-A-model!)

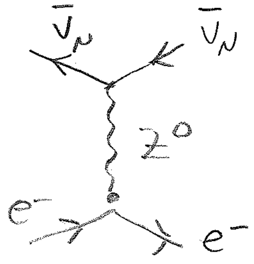
Thus,

$$\hat{\mathcal{L}}_I = i \frac{g_w^2}{2} J_0^\mu \frac{-i g_{\mu\nu}}{q^2 + m_Z^2} (J_0^\nu)^+$$

$m_Z \simeq 91.1876 \pm 0.0021$ GeV

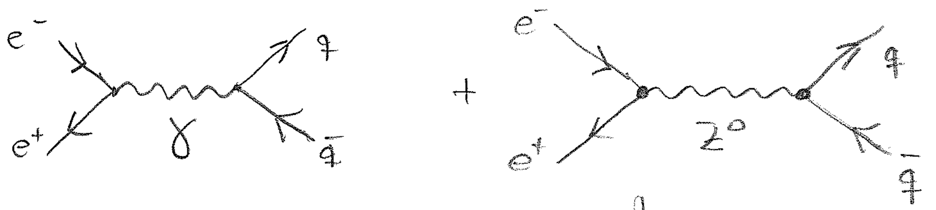
As opposed to W^+, W^-, Z^0 does not carry electric charge.

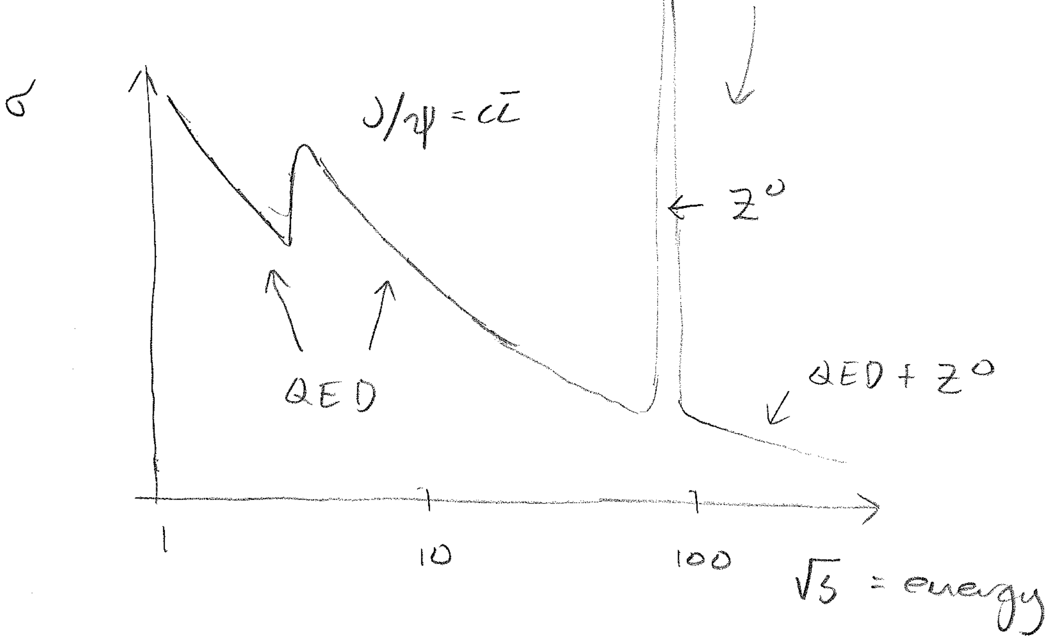
* observable in, e.g. $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ -scattering



("Neutral currents", CERN 1973)

* Clearly visible in $e^+e^- \to$ hadrons:





* Weak interactions actually are stronger than QED at $\sqrt{s} \gtrsim m_W, m_Z$!

The coupling $\alpha_W \equiv \frac{g_W^2}{4\pi} \simeq \frac{1}{29} > \frac{1}{139} = \alpha_{QED}$!

Weakness at low energies due to large m_W, m_Z

11. Standard model

* Fully renormalisable

* <u>Gauge</u> interactions : groups

$$U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$$

$SU(3)_c$: strong, colour $\begin{pmatrix} R \\ G \\ B \end{pmatrix}$

$SU(2)_L$: left-handed weak
$W^\pm, Z^0$ $\quad \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ - SU(2) doublets

$U(1)_Y$: Hypercharge $\rightarrow$ electromagnetism

* Matter (fermions)

Leptons: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$; e_R, μ_R, τ_R

Quarks: $\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L$; $u_R, d_R, \ldots$

* <u>Higgs particle</u> (scalar)

- Responsible for all quark and lepton masses, $W^\pm, Z^0$ masses + mixings
- these due to <u>coupling</u> <u>of quarks to Higgs</u>

+ <u>spontaneous symmetry breaking</u> of $SU(2)_L$

- Higgs is $SU(2)_L$ -doublet and has $U(1)_Y$ -charge

$\rightarrow$ makes W, Z - bosons massive

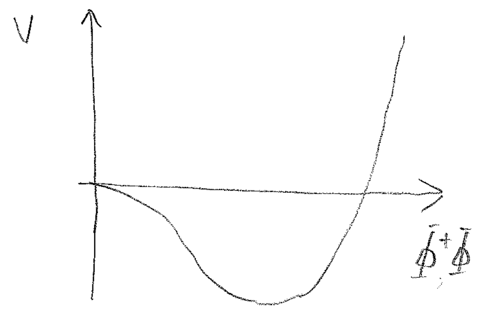
$\Rightarrow$ general ugliness in weak sector

* Higgs is essential & unique particle in S.M.
* Difficult to see experimentally, current mass limit $m_H \gtrsim 114$ GeV (CERN)

What is spontaneous symmetry breaking?

* Higgs has potential $V(\Phi) = -\mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2$

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \quad ; \quad \Phi_i \in \mathbb{C}$$

* 

- minimum at non-zero value of $|\Phi|$

$\Rightarrow$ Higgs acquires a non-zero expectation value $|\Phi| = \frac{1}{\sqrt{2}} v$. Without loss of generality, we can choose $\Phi \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ $\quad (v \in \mathbb{R})$

* let us consider Higgs coupling to $SU(2)_L$ only (in reality it also couples to $U(1)_Y$, which complicates things slightly)

Gauge invariance dictates that Higgs - SU(2) gauge boson coupling is through

$$\delta\mathcal{L} = [D_\mu \Phi]^\dagger [D^\mu \Phi]$$

$$D_\mu = \partial_\mu - i g_W A_\mu^a \frac{\sigma^a}{2}$$

$\nwarrow$ SU(2) generator, σ^a = Pauli matrices

$\uparrow$ SU(2) gauge, "W"

Thus, assuming constant $\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$,

$$\delta\mathcal{L} = \frac{1}{8}\left[i g_W A_\mu^a \sigma^a \begin{pmatrix} 0 \\ v \end{pmatrix}\right]^\dagger \left[i g_W A^{b\mu} \sigma^b \begin{pmatrix} 0 \\ v \end{pmatrix}\right]$$

$$= \frac{v^2}{8} g_W^2 A_\mu^a A^{b\mu} (0\ 1) \underbrace{\sigma^a \sigma^b}_{\mathbb{1}\delta^{ab} + i\epsilon^{abc}\sigma^c} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\nwarrow$ antisymm. in $a \leftrightarrow b$, vanishes

$$= \frac{v^2}{8} g_W^2 A_\mu^a A^{a\mu}$$

$$= \frac{1}{2} m_W^2 A_\mu^a A^{a\mu} \quad ; \quad \underline{m_W = \frac{1}{2} g_W v}$$

$\Rightarrow$ 3 massive gauge bosons, with $m = m_W = \frac{1}{2} g_W v$
$a = 1,2,3$

* In reality, due to coupling to $U(1)_Y$, one of these mixes with $U(1)_Y$ "photon" B:

$$\left.\begin{matrix} A^3 \\ B \end{matrix}\right\} \longrightarrow \begin{matrix} Z^0, \ m = m_Z \\ \gamma, \ m = 0 \end{matrix}$$

* Recall $\dfrac{g^2_W}{4\sqrt{2}\, m_W^2} = G_F \Rightarrow \underline{v^2 = \dfrac{1}{\sqrt{2}\, G_F}}$

Quark and lepton masses

Arise through couplings of type

$$\delta \mathcal{L} = -h_u \, (\bar{u} \ \bar{d})_L \, \tilde{\Phi} \, u_R + h.c.$$

$$-h_d \, (\bar{u} \ \bar{d})_L \, \Phi \, d_R + h.c$$

where $\tilde{\Phi} = i\sigma_2 \Phi^*$. Using $\Phi = \frac{1}{\sqrt{2}} \binom{0}{v}$, this gives

$$\delta \mathcal{L} = -\frac{h_u}{\sqrt{2}} \left(v \bar{u}_L u_R + v \bar{u}_R u_L \right) - \frac{h_d}{\sqrt{2}} \left(v \bar{d}_L d_R + v \bar{d}_R d_L \right)$$

$$= -\underbrace{\frac{h_u v}{\sqrt{2}}}_{= m_u} \bar{u} u - \underbrace{\frac{h_d v}{\sqrt{2}}}_{= m_d} \bar{d} d$$

h : Yukawa couplings. All masses depend linearly on v!

12 Current research

* Neutrino masses? $m_\nu = 0$ in classic standard model, but recent experiments show non-zero value.

  - Can be searched in precision measurements of β-decay, e.g.

    $$^3H \to {}^3He + e^- + \bar{\nu}_e$$

    - Mainz exp. (1997-2001) $m_\nu < 2.2\,eV$ upper limit
    - KATRIN (Karlsruhe) (2008→) $m_\nu < 0.2\,eV$

  - Neutrino oscillations show non-zero value: Like the Cabibbo-angle for quarks, weak interactions may generate a linear combination of mass eigenstates → oscillations

    - If ν_1, ν_2 are mass eigenstates, but weak interactions operate on $\nu_{1'}, \nu_{2'}$ which are linear combinations of ν_1, ν_2, then probability of oscillation

    $$P(1' \to 2') \propto \sin^2\left(\frac{L \cdot \Delta m^2}{4E}\right)$$

    L = distance from source, $\Delta m = m_{\nu_1} - m_{\nu_2}$

    E = energy

because typical $E \sim \text{GeV}$, $\Delta m \ll \text{eV}$,

we need long distances! E.G.

$$L \gtrsim \frac{4\,\text{GeV}}{\underset{\uparrow \atop \Delta m^2}{10^{-4}\,\text{eV}^2}} = 4 \cdot 10^{22} \frac{1}{\underbrace{\text{GeV}}_{\frac{1}{5}\text{fm}}} \simeq 10^{7}\,\text{m} = 1000\,\text{km}$$

Sources:
- Sun $(L \sim 150 \times 10^9\,\text{m})$
- Cosmic rays hitting the atmosphere on opposite side of earth $(L \sim 12\,000\,\text{km})$
- Accelerators $(L \sim 250\,\text{km}$, more planned$)$
- Nuclear reactors $(L \sim 10 - 500\,\text{km})$

Current interpretation:

$$|m_{\nu_\mu}^2 - m_{\nu_\tau}^2| \sim 2 \times 10^{-3}\,\text{eV}^2 \quad \text{(atmospheric)}$$

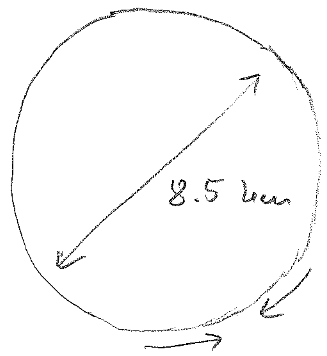
$$|m_{\nu_e}^2 - m_{\nu_\mu}^2| \text{ or } |m_{\nu_e}^2 - m_{\nu_\tau}^2| \sim 2 \times 10^{-5}\,\text{eV}^2$$

(solar)

---

* Future accelerator experiments

- LHC, CERN 2007
  - * Higgs boson!
  - * Physics beyond the standard Model ?
    - Supersymmetry, GUT, strings... (see page 22)
    - Huge amount of theoretical activity!



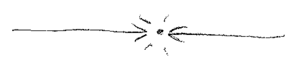
Counterrotating proton beams, energy 7 TeV/proton

or 3 TeV/nucleon, when using heavy nuclei (^{208}Pb)

Experiments: ATLAS, CMS, ALICE, LHC-B

Proposed:

ILC International linear collider
e^+e^-, 0.5–1.0 TeV
(cleaner as LHC, because $p \to e$!)
Linear, length ~ 20 km?



CLIC e^+e^-, 3–5 TeV, linear
advanced technology; 2020?

VLHC pp, 60 TeV, d ~ 20 km (?)

$\mu^+\mu^-$ 0.5–3 TeV ?

- Protons heavy, easier to reach high energy in rings. Collisions messy because of extra partons
- Electrons light → synchrotron losses in rings → linear colliders. Clean reaction
- μ heavy & clean. However decays quickly!

* Cosmology (page 24)

- Many problems not explained by S.M. !

* Dark matter (exotic particles)

* Dark energy (vacuum energy; exotic fields)

* Why there is matter at all ?

- Lot of information about new physics

- Precision measurement : cosmic microwave background

Hauskaa kevättä & kesää !

Have a nice spring & summer !