- Thus, u, d -quarks form <u>isospin</u> doublet

$I = \frac{1}{2}$, $I_3(u) = +\frac{1}{2}$, $I_3(d) = -\frac{1}{2}$ (j, m)

- Addition as with angular momenta.

- Example: baryons of u,d -quarks

$$2 \otimes 2 \otimes 2 = 2 \oplus 2 \oplus 4$$

Another of 2's contain p and n:

p	(uud)	$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$
n	(udd)	$\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$

$m_p \simeq m_n$

And, we find the following 4 Δ-baryons

Δ^{++}	(uuu)	$\left|\frac{3}{2}\ \frac{3}{2}\right\rangle$
Δ^{+}	(uud)	$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle$
Δ^{0}	(udd)	$\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle$
Δ^{-}	(ddd)	$\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle$

$I = \frac{3}{2}$ -rep., with $m_\Delta \simeq 1231-1234$ MeV!

- Great success! masses really degenerate <u>within</u> a <u>representation</u>.

NOTE: Isospin is a symmetry of <u>strong</u> interactions (approximate, because $m_u \neq m_d$).

Masses of baryons caused by strong interact.

Historically, the symmetry $p \Leftrightarrow n$ was called isospin; i.e. p and n belong to the same isospin doublet.

Isospin for antiparticles

Antiquarks $\bar{u}, \bar{d}$ also belong to $I = \frac{1}{2}$, but I_3 is opposite: thus, if we transform isodoublet

$$\begin{pmatrix} u \\ d \end{pmatrix} \to \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \begin{matrix} \leftarrow -\frac{1}{2} \\ \leftarrow \frac{1}{2} \end{matrix} \Bigg\} I_3$$

we should rotate this further in isospin-space

$$U \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} = e^{-i\pi\sigma_2/2} \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} = \left(\mathbb{1} \cos\frac{\pi}{2} - i\sigma_2 \sin\frac{\pi}{2} \right) \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} = \underline{\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}}$$

Thus, we take $\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$ to be the anti-u,d, isospin doublet, i.e.

$$\bar{u} = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle, \quad \bar{d} = -|\tfrac{1}{2}\ \tfrac{1}{2}\rangle$$

- Thus, combining u, d with $\bar{u}, \bar{d}$, we get mesons (compare p. 44)

$$
\left.\begin{array}{lll}
|1\,1\rangle = \pi^+ = u\bar{d} & m = 139.6 \text{ MeV} & \\
|1\,0\rangle = \pi^0 = \frac{1}{\sqrt{2}}(u\bar{d} - d\bar{u}) & m = 135.0 & \\
|1\,{-1}\rangle = \pi^+ = d\bar{u} & m = 139.6 &
\end{array}\right\} I = 1
$$

sign

$$
|0\,0\rangle = \eta^* = \frac{1}{\sqrt{2}}(u\bar{d} + d\bar{u}) \qquad m \sim 550 \qquad I = 0
$$

does not exist as such
mixes with $s\bar{s}$-state!

Again, symmetry works well!

- Applies also for <u>nuclear physics</u>:

$$
\left.\begin{array}{l}
|1\,1\rangle = pp \\
|1\,0\rangle = \frac{1}{\sqrt{2}}(pn + np) \\
|1\,{-1}\rangle = nn
\end{array}\right\} I = 1
$$

$$
|0\,0\rangle = \frac{1}{\sqrt{2}}(pn - np) \qquad I = 0
$$

Only $I=0$ state forms bound state, deuterium.

$\Rightarrow$ attraction in $I=0$ channel, repulsion in $I=1$

• Application to baryon-baryon scattering (<u>only</u> applies to strong interactions)

a) $p + p \rightarrow d + \pi^+$: $|11\rangle \rightarrow |11\rangle$

b) $p + n \rightarrow d + \pi^0$ $\frac{1}{\sqrt{2}}(|10\rangle + |00\rangle) \rightarrow |10\rangle$

c) $n + n \rightarrow d + \pi^-$ $|1-1\rangle \rightarrow |1-1\rangle$

$\uparrow$ deut., $I = 0$

($p + n = \frac{1}{\sqrt{2}}(|10\rangle + |00\rangle)$ from Clebsch-Gordans!)

$\Rightarrow$ scattering amplitudes

$$M_a : M_b : M_c = 1 : \frac{1}{\sqrt{2}} : 1$$

$\Rightarrow$ cross-sections (probabilities)

$$\sigma_a : \sigma_b : \sigma_c = \underline{1 : \frac{1}{2} : 1}$$

Experimentally works well

• Larger flavour groups

- What if $m_u = m_d = m_s$? Then the isospin $SU(2)$ symmetry $\Rightarrow SU(3)$, i.e.

$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = U \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

would be a symmetry of strong interactions!

- broken in reality by $m_s \gg m_u, m_d$
$\Rightarrow$ symmetry approximate

- To be discussed in next section

- likewise, $c, b, t \Rightarrow SU(4), SU(5), SU(6)$

broken by m_q

$SU(4)$: mediocre, $SU(5)$: bad, $SU(6)$: useless

- NOTE: by definition, Isospin $\neq 0$ only for u, d! other quarks (and leptons) have $I = 0$.

3.7. C, P, T symmetries

3.7.1. Parity P

- reflection $\bar{r} \to -\bar{r}$ (or $\bar{x}_1 \to -\bar{x}_1$, say) reflection
- 2 reflections: $\bar{r} \to \bar{r}$, i.e.

$$P^2 = 1$$

$\Rightarrow$ eigenvalues of $\underline{P = \pm 1}$

- Particles have intrinsic parity P_i
- Orbital wave functions have $P = (-1)^\ell$

where $\psi(\bar{r}) = R(r)\, Y_{\ell m}(\hat{r}) = (-1)^\ell R(r)\, Y_{\ell m}(-\hat{r})$

- Thus, a 2-particle eigenstate of angular momentum has parity

$$P = P_1 \cdot P_2 \cdot (-1)^\ell$$

- Parity is multiplicative!

- Different scalars and vectors: let $\vec{A}$, $\vec{B}$ be ordinary vectors, $P\vec{A} = -\vec{A}$

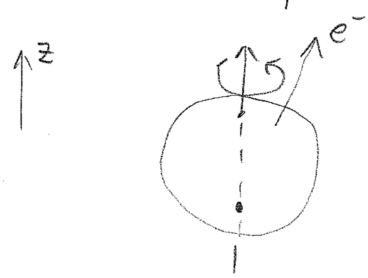
- Scalar s: $Ps = s$ e.g. $\vec{A} \cdot \vec{B}$
- Vector $\vec{v}$: $P\vec{v} = -\vec{v}$ e.g. $\vec{A}$, $\vec{B}$
- Pseudovector $\vec{p}$: $P\vec{p} = \vec{p}$ e.g. $\vec{p} = \vec{A} \times \vec{B}$
- Pseudoscalar p: $Pp = -p$ e.g. $p = \bar{A} \times \bar{B} \cdot \bar{C}$

(Examples: angular momentum $\bar{J} = \bar{r} \times \bar{p}$, magnetic field $\bar{B}$ are pseudovectors; $\bar{V} \cdot \bar{J}$ is pseudoscalar)

- Strong and EM interactions conserve parity; <u>weak violate</u> it!

- Before 1956 symmetry under parity was "self-evident"; 1956 Lee and Yang suggested trying to find P-violation in weak interactions. Found by C.S. Wu:

^{60}Co β-decay, $n \to p + e + \bar{\nu}_e$



- spin of ^{60}Co along z-axis
- electron comes off towards positive z-direction!
- spin $\bar{s}$ pseudovector, $\bar{v}_e$ vector
  $\Rightarrow$ in reflection $\bar{s}$ does not change, but $\bar{v}_e$ points down!

$\Rightarrow$ P violation

$\Rightarrow$ huge impact to fundamental physics!

- What happens? Weak interactions ($W^\pm$, Z) couple only to <u>left-handed particles</u> or <u>right-handed antiparticles</u>