SU(2) rep. 4 $(j=\frac{3}{2})$:

$$|\tfrac{3}{2}\;\tfrac{3}{2}\rangle = \uparrow\uparrow\uparrow$$

$$|\tfrac{3}{2}\;\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$$

$$|\tfrac{3}{2}\;-\tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow)$$

$$|\tfrac{3}{2}\;-\tfrac{3}{2}\rangle = \downarrow\downarrow\downarrow$$

Thus, from pages 60, 64 and above we see that for example Δ^0-spin $+\frac{1}{2}$-state

$$|\Delta^0;\tfrac{1}{2}\rangle = (udd)_{symm.} \times (\uparrow\uparrow\downarrow)_{symm.}$$

$$= \tfrac{1}{\sqrt{3}}(udd + dud + ddu)\,\tfrac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$$

or Ω^- - spin $-\frac{3}{2}$

$$|\Omega^-;-\tfrac{3}{2}\rangle = (sss)\times(\downarrow\downarrow\downarrow)$$ (each s-quark spin is $\downarrow$)

For baryons in octet situation is more cumbersome due to the mixed symmetry. The result is that the total flavor-spin wave function is not a direct product $(8_{Flav}\otimes 2_{spin})$, but we need to combine flavor-spin combinations to obtain a symmetric state

Thus, combining antisymmetric parts:
let abc - quark labels, $s_1 s_2 s_3$ - spin indices,
then

$$\psi^{\text{octet}} = N \cdot \Big[\tfrac{1}{2}(ab - ba)c\,(s_1 s_2 - s_2 s_1) s_3$$
$$+ \tfrac{1}{2} a(bc - cb)\, s_1 (s_2 s_3 - s_3 s_2)$$
$$+ \tfrac{1}{2}(abc - cba)\,(s_1 s_2 s_3 - s_3 s_2 s_1)\Big] + \begin{matrix}\text{cyclic}\\ \text{permutations}\end{matrix}$$

Thus, proton in spin $+\frac{1}{2}$ state: uud, $\uparrow\uparrow\downarrow$

$$|p:\uparrow\rangle = \frac{N}{2}\Big[(uud - udu)(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)$$
$$+ (uud - duu)(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)$$
$$+ (udu - duu)(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)\Big]$$

$$= \frac{1}{3\sqrt{2}}\Big[uud\,(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$
$$+ udu\,(2\uparrow\downarrow\uparrow - \uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow)$$
$$+ duu\,(2\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)\Big]$$

$$\frac{1}{\sqrt{(4+1+1)\cdot 3}}$$

Thus, in $s = +\frac{1}{2}$ proton d-quark is twice as likely to be $\downarrow$ than $\uparrow$!
($s = -\frac{1}{2}$ opposite, naturally)

- It is possible to write down similar mass formula for baryons as for mesons, with good results (Griffiths p. 184)

- Likewise, the <u>magnetic moment</u> μ of $\ell = 0$ baryons can be calculated in terms of quark magnetic moments

point s, ½ particle $\searrow$ $\vec{\mu} = \frac{q}{m}\vec{S} \Rightarrow$ z-component $\underline{\mu = \frac{q}{2m}}$ $(s = +\frac{1}{2})$

$$\Rightarrow \mu_u = \frac{2}{3}\frac{1}{2m_u} \; ; \; \mu_d = -\frac{1}{3}\frac{1}{2m_d} \; ; \; \mu_s = -\frac{1}{3}\frac{1}{2m_s}$$

$$\Rightarrow \mu_B = \langle B\uparrow | (\mu_z^1 + \mu_z^2 + \mu_z^3) | B\uparrow \rangle$$

$$= 2 \langle B\uparrow | \sum_{i=1}^{3} \mu_i S_z^{(i)} | B\uparrow \rangle \qquad (S_z = \pm\tfrac{1}{2})$$

$$= \left(\frac{1}{3\sqrt{2}}\right)^2 \left(4(\mu_u + \mu_u - \mu_d) + 2(\mu_u - \mu_u + \mu_d)\right) \times 3$$

$$= \underline{\frac{1}{3}(4\mu_u - \mu_d)}$$

- Likewise for the whole octet

$$M = m_1 + m_2 + m_3 + A' \left[\frac{\bar{S}_1 \cdot \bar{S}_2}{m_1 m_2} + \frac{\bar{S}_1 \cdot \bar{S}_3}{m_1 m_3} + \frac{\bar{S}_2 \cdot \bar{S}_3}{m_2 m_3} \right]$$

TABLE 5.6 BARYON OCTET AND DECUPLET MASSES (MeV/c^2)

Baryon	Calculated	Observed
N	939	939
Λ	1116	1114
Σ	1179	1193
Ξ	1327	1318
Δ	1239	1232
Σ^*	1381	1384
Ξ^*	1529	1533
Ω	1682	1672

Griffiths, p. 182

TABLE 5.5 MAGNETIC DIPOLE MOMENTS OF OCTET BARYONS

Baryon	Moment	Prediction	Experiment
p	$(\frac{4}{3})\mu_u - (\frac{1}{3})\mu_d$	2.79	2.793
n	$(\frac{4}{3})\mu_d - (\frac{1}{3})\mu_u$	−1.86	−1.913
Λ	μ_s	−0.58	−0.61
Σ^+	$(\frac{4}{3})\mu_u - (\frac{1}{3})\mu_s$	2.68	2.33 ± 0.13
Σ^0	$(\frac{2}{3})(\mu_u + \mu_d) - (\frac{1}{3})\mu_s$	0.82	
Σ^-	$(\frac{4}{3})\mu_d - (\frac{1}{3})\mu_s$	−1.05	−1.41 ± 0.25
Ξ^0	$(\frac{4}{3})\mu_s - (\frac{1}{3})\mu_u$	−1.40	−1.253 ± 0.014
Ξ^-	$(\frac{4}{3})\mu_s - (\frac{1}{3})\mu_d$	−0.47	−0.69 ± 0.04

The numerical values are given as multiples of the nuclear magneton, $e\hbar/2m_pc = 3.152 \times 10^{-18}$ MeV/gauss.
Source: S. Gasiorowicz and J. L. Rosner, *Am. J. Phys.* **49**, 954 (1981).

NOTE: These calculations by no means constitute <u>solving</u> hadron structure. (approximate) Symmetries were only used to give <u>ratios</u> of some quantities or rough estimates. To really solve the structure of proton (from QCD) requires large computer simulations (lattice QCD).

3.8 C, P, T symmetries

3.8.1. Parity P

- reflection $\bar{r} \to -\bar{r}$ (or $\bar{x}_1 \to -\bar{x}_1$, say) reflection
- 2 reflections: $\bar{r} \to \bar{r}$, i.e.

$$P^2 = 1$$

$\Rightarrow$ eigenvalues of $\underline{P = \pm 1}$

- Particles have <u>intrinsic</u> parity P_i
- Orbital wave functions have $\underline{P = (-1)^l}$

where $\psi(\bar{r}) = \underline{R(r)\, Y_{lm}(\hat{r}) = (-1)^l R(r)\, Y_{lm}(-\hat{r})}$

- Thus, a 2-particle eigenstate of angular momentum has parity

$$\underline{P = P_1 \cdot P_2 \cdot (-1)^l}$$

- Parity is <u>multiplicative</u>!

- <u>Different scalars and vectors</u>: let $\vec{A}$, $\vec{B}$ be ordinary vectors, $P\vec{A} = -\vec{A}$

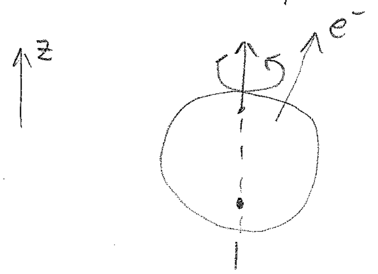
- Scalar s: $Ps = s$ e.g. $\vec{A} \cdot \vec{B}$
- Vector $\vec{v}$: $P\vec{v} = -\vec{v}$ e.g. $\vec{A}$, $\vec{B}$
- Pseudovector $\vec{p}$: $P\vec{p} = \vec{p}$ e.g. $\underline{\vec{p} = \vec{A} \times \vec{B}}$
- Pseudoscalar p: $Pp = -p$ e.g. $p = \bar{A} \times \bar{B} \cdot \bar{C}$

(Examples: angular momentum $\bar{J} = \bar{r} \times \bar{p}$,
magnetic field $\bar{B}$ are pseudovectors;
$\bar{V} \cdot \bar{J}$ is pseudoscalar)

- Strong and EM interactions conserve parity; <u>weak violate</u> it!

- Before 1956 symmetry under parity was "self-evident"; 1956 Lee and Yang suggested trying to find P-violation in weak interactions. Found by C.S.Wu:

^{60}Co β-decay, $n \to p + e + \bar{\nu}_e$



- spin of ^{60}Co along z-axis
- electron comes off towards positive z-direction!
- spin $\bar{s}$ pseudovector, $\vec{\nu}_e$ vector
  $\Rightarrow$ in reflection $\vec{s}$ does not change, but $\vec{\nu}_e$ points down!

$\Rightarrow$ P violation

$\Rightarrow$ huge impact to fundamental physics!

$$\boxed{\begin{array}{l}\bullet \text{ What happens? Weak interactions } (W^{\pm}, Z) \\ \text{couple only to } \underline{\text{left-handed}}\ \underline{\text{particles}} \\ \text{or } \underline{\text{right-handed}}\ \underline{\text{antiparticles}}\end{array}}$$

For massless particles, left handed = left helicity

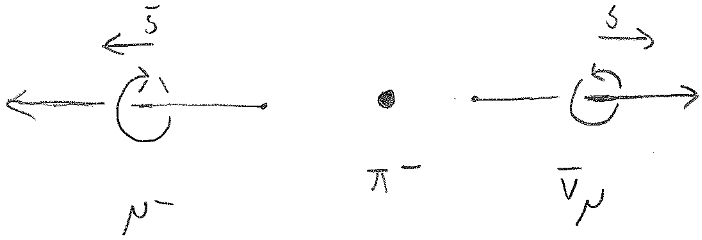
right-handed (helicity $m_s = +\frac{1}{2}$) left-handed ($m_s = -\frac{1}{2}$)

- In normal Standard Model neutrinos are left handed; antineutrinos right-handed!

- (when $m_\nu \neq 0$, right-handed ν exist too)

- For massive particles, we can always switch frame so that left helicity $\Leftrightarrow$ right helicity! (boosting past the particle)

- Experimental signal: decay of π^- at rest $\pi^- \to \mu^- + \bar{\nu}_\mu$ (or $\pi^+ \to \bar{\mu} + \nu_\mu$)



- μ^- always comes out right-handed
  $\Rightarrow \bar{\nu}_\mu$ right-handed

- On the other hand, $\pi^0 \to \gamma + \gamma$
  the photons have left/right helicity with 50 % probability!

Parity of S.M. particles and hadrons

- QFT indicates that antiparticles have following P:
  - Fermion f: $\underline{P_{\bar{f}} = -P_f}$
  - Boson b: $\underline{P_{\bar{b}} = P_b}$

- We take: $\begin{cases} P_{quark} = +1 \\ P_{antiquark} = -1 \end{cases}$ (convention)

Thus:

(j=0) $\underline{l=0,\ s=0 \text{ mesons}}$: $\underline{P} = P_q \cdot P_{\bar{q}} \cdot (-1)^{l \overset{"}{=} 0} = \underline{-1}$

$\vec{J} = \vec{S} + \vec{L}$ $\underline{\text{pseudoscalars}}$ π, K, η, η'

(j=1) $\underline{l=0,\ s=1 \text{ mesons}}$: $\underline{P = -1}$

$\underline{\text{vector mesons}}$ ρ, K^*, ω, ϕ

(j=1) $\underline{l=1,\ s=0 \text{ mesons}}$: $\underline{P=1}$ $\underline{\text{pseudovectors}}$

(j=½) $\underline{l=0,\ s=\tfrac{1}{2}}$ $\begin{cases} \text{baryons: } \underline{P = 1^3 = 1} & (p, n, \ldots) \\ \text{antibaryons: } \underline{P = -1} & (\bar{p}, \bar{n}, \ldots) \end{cases}$

- photon γ, gluon g: vectors, $\underline{P = -1}$
- ν - not a parity eigenstate! (nor W, Z)

3.8.2 Charge conjugation $\mathbb{C}$

- particle $\leftrightarrow$ antiparticle

$$C p = \bar{p} \quad ; \quad C \bar{p} = p$$

$\Rightarrow C^2 = 1 \quad \Rightarrow$ eigenvalues ± 1

- Only particles which are own antiparticles can be eigenstates of C
  - photon γ: $\quad C\gamma = -\gamma \quad (e \to -e)$
  - $q\bar{q}$ - mesons: $\quad C(q\bar{q}) = (-1)^{\ell+s}(q\bar{q})$

same where ℓ - angular momentum, s - spin
($\pi_0, \eta, \eta', \omega, \phi, \dots$)

- Multiplicative quantum number

$\boxed{\text{• } C \text{ conserved in strong, EM interactions, violated in Weak}}$

- Only rarely applicable, because only few particles eigenstates of C
- Example: $\quad \pi^0 \to \gamma + \gamma \quad (C: 1 \to (-1)^2 = 1)$

$\underline{\text{NOT}} \quad \pi^0 \to \gamma + \gamma + \gamma \quad C: 1 \to (-1)^3$

- Notation: for mesons,

J^{PC}: 0^{-+} $\quad \ell=0, s=0 \quad (\pi, K, \eta, \eta')$

1^{--} $\quad \ell=0, s=1 \quad (\rho, K^* \ldots)$

1^{+-} $\quad \ell=1, s=0$

$$\left.\begin{matrix} 0^{++} \\ 1^{++} \\ 2^{++} \end{matrix}\right\} \ell=1, s=1, \quad \begin{matrix} j=0 \\ j=1 \\ j=2 \end{matrix}$$

here full nonet is labeled with C of the center.

3.8.3 CP violation

- After P violation was found, it was thought that the combined CP is exact.

Example

$$\pi^+ \to \mu^+ + \nu_\mu \qquad (\mu^+, \nu_\mu \text{ left-handed})$$

$C \downarrow$

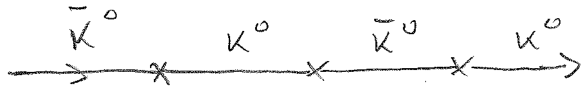
$$\pi^- \to \mu^- + \bar{\nu}_\mu \qquad \mu^-, \bar{\nu}_\mu \text{ left – DOES NOT EXIST!}$$

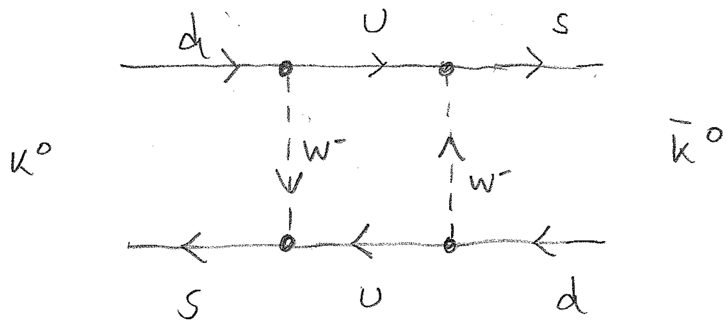
$P \downarrow$

$$\pi^- \to \mu^- + \bar{\nu}_\mu \qquad \mu^-, \bar{\nu}_\mu \text{ right-handed OK.}$$

However, <u>weak</u> interactions <u>violate CP</u> too! Found 1964 Cronin & Fitch

- Experiment: neutral Kaon beam
  - K^0-meson $(d\bar{s})$ can turn into $\bar{K}^0$ in flight: (only $D^0 \leftrightarrow \bar{D}^0$; $B^0 \leftrightarrow \bar{B}^0$ can do this!)





+ other diagrams

- Thus, what propagates is a linear combination of $K^0, \bar{K}^0$. We can form eigenstates of CP:

$$P|K^0\rangle = -|K^0\rangle \quad ; \quad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

$$C|K^0\rangle = |\bar{K}^0\rangle \quad ; \quad C|\bar{K}^0\rangle = |K^0\rangle$$

$$\Rightarrow CP|K^0\rangle = -|\bar{K}^0\rangle \; ; \; CP|\bar{K}^0\rangle = -|K^0\rangle$$

$\Rightarrow$ eigenstates of CP

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$CP|K_1\rangle = |K_1\rangle$$

$$CP|K_2\rangle = -|K_2\rangle$$

- Thus, if CP is fully conserved, decays are

$K_1 \rightarrow$ state with $CP = +1$

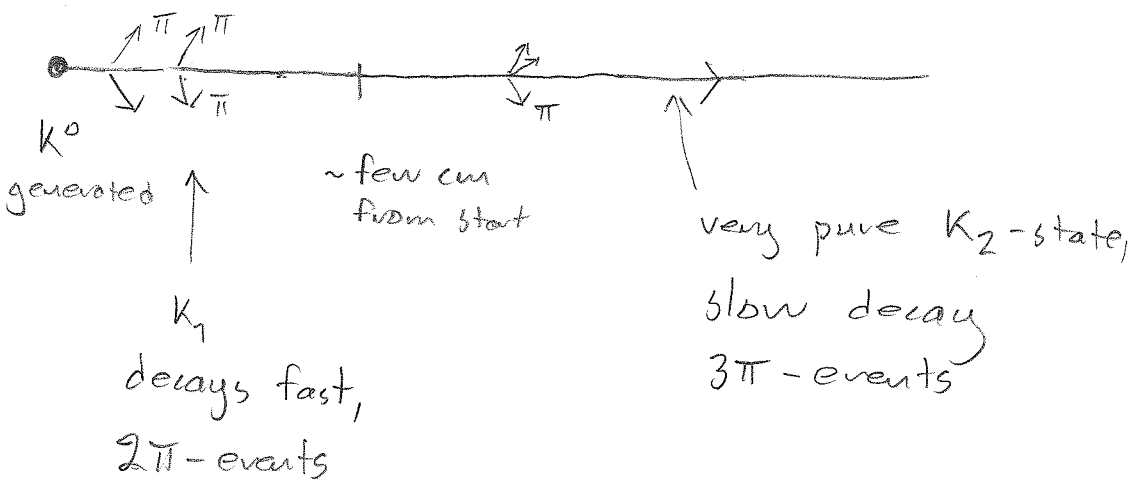
$K_2 \rightarrow$ state with $CP = -1$

Dominant decay: pions π ; $CP = -1$ $\Rightarrow$

$K_1 \rightarrow 2\pi$ fast (phase space, energy)

$K_2 \rightarrow 3\pi$ slow

Thus, if we produce $|K^0\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle)$,
K_1-component dies out quickly, leaving K_2:



- Proposed 1954 Gell-Mann, Pais; observed 1956 Lederman

Lifetimes:

$$\tau_{K_1} \simeq 0.9 \times 10^{-10}\ \text{s}$$

$$\tau_{K_2} \simeq 5.2 \times 10^{-8}\ \text{s}$$

- 1964 Cronin & Fitch observed that the pure K_2-beam can occasionally decay $\rightarrow 2\pi$, violating CP.
(~ 0.2 % of decays)

- Thus, the long-lived particle is actually a mixture of K_1 and K_2

$$|K_L\rangle \propto |K_2\rangle + \epsilon |K_1\rangle , \qquad \epsilon \simeq 2.3 \times 10^{-3}$$

- (here K_1 decays quickly, but CP-violation transforms some $K_2 \to K_1$, maintaining small mixture)

- $\cancel{CP}$ seen recently also in $B^0 - \bar{B}^0$ -system ($d\bar{b} \leftrightarrow b\bar{d}$) ; BaBar (Stanford), Belle (KEK, Japan)

  2001

- $C\cancel{P}$ finally destroys mirror symmetry
- $\cancel{C}P$ destroys symmetry between matter $\Leftrightarrow$ antimatter
  $\to$ baryogenesis, origin of matter in the early Universe

- Origin of $\cancel{CP}$: complex quark mass matrix, Cabibbo-Kobayashi-Maskawa (CKM)
  - Need <u>at least 3</u> quark families to have $\cancel{CP}$!
    physical in S.M.

3.8.4 Time reversal T $t \to -t$

- Combined <u>CPT</u> is exact in all reasonable theories! Thus, T violation ~ CP violation.
- Violation of CPT $\to$ violation of Lorentz invariance

Summary: symmetries

- Poincaré-symmetry (Lorentz + translations)
- "Gauge symmetries", local → Dynamics!
  - Color $SU(3)$ → QCD
  - Weak $SU(2)$ → weak
  - EM $U(1)$ → QED
  - All interactions in S.M. due to these gauge symmetries!
  - Exact & absolutely central
  - We have not discussed these yet (topic of Quantum Field Theory)
- Global symmetries ("accidental")
  - Baryon number B, lepton numbers L_i

    $\boxed{\text{These are consequence of } U(1)\text{-symmetries:}}$

    - Consider arbitrary process

      $\langle q_1' q_2' \cdots q_m' | S | q_1 q_2 \cdots q_n \rangle$
    - Multiply each $|q\rangle$ by $e^{i\theta}$, $\langle q'|$ by $e^{-i\theta}$, any θ
    - Require process does not change

      $\Rightarrow m = n$, i.e. number of q's constant

      $\Rightarrow B$ conserved
  - Flavour symmetries (approximate) ($SU(2)$, $SU(3)$, $SU(4)$, ...)
- Discrete symmetries

  P, CP, CPT

  violated by weak