1. Let's consider a uniform stretched string initially at rest. The string is flicked at point $x = a$, after which the system will evolve according to the equations of motions derived from the Lagrangian. Let's assume, that the perturbation happens at time $t = 0$, and can be approximated as
$$\dot{u}(x, 0) = v_0 \delta(x - a).$$
Write down the function $u(x, t)$, which gives us the oscillation amplitudes at all eigenfrequencies.

2. Go through the derivation of the Lagrange equation given in lectures (equation (93))
$$\frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial(\partial u/\partial t)} + \sum_i \frac{\partial}{\partial x_i}\frac{\partial \mathcal{L}}{\partial(\partial u/\partial x_i)} - \frac{\partial \mathcal{L}}{\partial u} = 0,$$
and show that the boundary condition (equation (92) in lectures)
$$\sum_i n_i \frac{\partial \mathcal{L}}{\partial(\partial u/\partial x_i)}\delta u = 0$$
should be required everywhere on the boundary of $V$ to get the result. Note that the parameters $n_i$ are the components of the unit vector $\hat{n}$ perpendicular to the surface element at a given point of the boundary of $V$.

3. Let's consider the functional
$$F(u) = \frac{1}{2}\int dx \int dy (\nabla u \cdot \nabla u + \lambda u^2)$$
with a given boundary conditions. Find out the differential equation, which function $u(x, y)$ must satisfy to give a minimum for the functional $F(u)$.
Discretize the functional $F(u)$ using points $(x_i, y_j) = (ai, aj)$. Using the discrete version $F(u_{ij})$ (short-hand notation $F(u_{ij})$ is used to mean that $F$ is a function of all independent variables $u_{ij}$), derive the equation, which the parameters $u_{ij}$ must satisfy to give a minimum for function $F(u_{ij})$.

4. Consider the functional of the previous problem with $\lambda = 0$. Let's define a current $\vec{j}$ to be
$$\vec{j} = \nabla u(x, y).$$

Show that

$$\nabla \cdot \vec{j} = 0,$$

if $u(x, y)$ satisfies the differential equation got in the previous problem. Transform the current to discrete lattice in the way, that the sum of all currents coming to a given lattice point is zero (assuming that $u_{ij}$ satisfies the discrete equation got in the previous problem).