

1. Let's study a situation, where an inertial coordinate frame K' is moving in x -direction with velocity v relative to another inertial coordinate frame K . Let's assume that a particle is moving with velocity \vec{u}' in the coordinate frame K' .

Derive how the 3-velocity \vec{u}' transforms in the Lorentz transformation to the coordinate frame K . (Hint: Write the Lorentz transformation for differentials dx, dy, dz and dt .)

Study how the energy E and momentum \vec{p} change in the Lorentz transformation.

Assume, that $v \ll c$ and $v' \ll c$. Show that with these approximations the energy and momentum transform under Lorentz transformation in the same way as they transform in the non-relativistic mechanics (note that the rest energy of the particle must be taken into account).

2. Use Lorentz transformation to show that
 - a) the velocity between two inertial coordinate frames is independent of which frame we choose to be moving relatively to the other (if the sign of the velocity is ignored).
 - b) if two events are timelike ($s^2 > 0$), it is always possible to find a coordinate frame, where they happen at the same place.
 - c) if two events are spacelike ($s^2 < 0$), it is always possible to find coordinate frame, where they happen simultaneously.
3. Let's consider how 4-dimensional volume element transforms under Lorentz transformation. The connection between small volume elements of two 4-dimensional coordinate systems can be expressed as $dx^0 dx^1 dx^2 dx^3 = J dx'^0 dx'^1 dx'^2 dx'^3$, where J is the Jacobian determinant of the transformation. Show that for Lorentz transformation $J = 1$.
4. In the context of formula (153) in lectures there was a short discussion, whether the action S can be chosen differently for free particles. Study what happens, if we define

$$S = \alpha \int_a^b u_i dx^i.$$