1. Let's consider a static magnetic field $\mathbf{B}$ introduced in lectures (charges causing the magnetic field move only within a restricted region and the static magnetic field $\mathbf{B}$ is an average of the real magnetic field over long period of time). In this case we get from Maxwell's equations

$$\begin{aligned} \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{j}, \end{aligned}$$

and in lectures it was stated that the vector potential satisfying the equations is

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int dV' \frac{\mathbf{j}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}.$$

Show that the vector potential $\mathbf{A}$ satisfies the Coulomb gauge condition

$$\nabla \cdot \mathbf{A} = 0.$$

2. Go through section "plane wave" in the lecture notes and calculate in particular the energy density, Poynting vector and Maxwell's stress tensor for a plane wave.

3. Go through section "Fourier transform of a static field" in the lecture notes, and show in particular that the inverse Fourier transform can be calculated from the formula

$$f(\mathbf{r}) = \frac{1}{V} \sum_{\mathbf{k}} f_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}.$$