1. Fourier transform over both time and space is defined as

$$F(\mathbf{k},\omega) = \int dt \int dV f(\mathbf{r},t) e^{-i\mathbf{k}\cdot\mathbf{r}} e^{i\omega t}.$$

Write down the inverse continuous Fourier transform (the transform in the case, when $L_x, L_y, L_z \to \infty$).
Note that the delta-function can in the continuos case be written as

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iyx} dy.$$

Use fourier transform directly to differential equation

$$\left(\nabla_r^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G(\mathbf{r},t) = -\delta(\mathbf{r})\delta(t)$$

defining the Green's function G to show that

$$G(\mathbf{k},\omega) = \frac{1}{k^2 - \omega^2/c^2}.$$

Fourier transform the Green's function

$$G(\mathbf{r},t) = \frac{1}{4\pi r}\delta(t - r/c)$$

obtained in lectures to show that this gives the same result.
Hint: You may have to include a damping factor

$$\lim_{\epsilon\to 0} e^{-\epsilon r}$$

to get the integrals converging. This can be interpreted as effectively taking into account that the space is in fact only finite (even though we let the size of the box to go infinite as a limiting value), since the damping will only contribute to the function at 'large values of r'.

2. Show that the 4-vector form of the Lienard-Wiechert potential

$$A^i(\mathbf{r},t) = \frac{e}{4\pi\epsilon_0}\frac{u^i}{cR_k u^k}$$

gives the equations

$$\phi(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\frac{e}{R}, \qquad \mathbf{A}(\mathbf{r},t) = 0$$

in the coordinate system, where the particle is at rest.
Show also that in the general coordinate system, where the particle moves with velocity $\mathbf{v}$, the 4-vector form gives equations

$$\phi(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\frac{e}{R - \mathbf{R}\cdot\mathbf{v}/c}, \qquad \mathbf{A}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\frac{e\mathbf{v}}{c^2(R - \mathbf{R}\cdot\mathbf{v}/c)}.$$