1. Let's study a situation, where an inertial coordinate frame K' is moving in x-direction with velocity v relative to another inertial coordinate frame K. Let's assume that a particle is moving with velocity $\vec{u'}$ in the coordinate frame K'.
   Derive how the 3-velocity $\vec{u'}$ transforms in the Lorentz transformation to the coordinate frame K. (Hint: Write the Lorenz transformation for differentials dx, dy, dz and dt.)
   Study how the energy E and momentum $\vec{p}$ change in the Lorentz transformation.
   Assume, that $v \ll c$ and $v' \ll c$. Show that with these approximations the energy and momentum transform under Lorentz transformation in the same way as they transform in the non-relativistic mechanics (note that the rest energy of the particle must be taken into account).

2. Use Lorentz transformation to show that

   a) the velocity between two inertial coordinate frames is independent of which frame we choose to be moving relatively to the other (if the sign of the velocity is ignored).

   b) if two events are timelike ($s^2 > 0$), it is always possible to find a coordinate frame, where they happen at the same place.

   c) if two events are spacelike ($s^2 < 0$), it is always possible to find coordinate frame, where they happen simultaneously.

3. Let's consider how 4-dimensional volume element transforms under Lorentz transformation. The connection between small volume elements of two 4-dimensional coordinate systems can be expressed as $dx^0dx^1dx^2dx^3 = Jdx'^0dx'^1dx'^2dx'^3$, where J is the Jacobian determinant of the transformation. Show that for Lorentz transformation $J = 1$.

4. In the context of formula (156) in lectures there was a short discussion, whether the action S can be chosen differently for free particles. Study what happens, if we define

$$S = \alpha \int_a^b u_i dx^i.$$