1. Let's consider a charged spatial non-relativistic harmonic oscillator with mass m and charge e, which can oscillate in any direction i.e. a force $\mathbf{F} = -k\mathbf{r}$ is applied to it, when no electric or magnetic fields are present (origo is set in the equilibrium position of the particle).
   The angular frequency of the vibrations when no electric or magnetic fields are present is of course $\omega_0 = \sqrt{k/m}$.
   Assume now that the oscillator is in a constant uniform magnetic field $\mathbf{B} = B\mathbf{k}$. Determine the possible values for angular frequency ω of vibration in the (x, y)-plane, and show that for weak fields $|\omega| = \omega_0 \pm \frac{eB}{2m}$.

2. In lectures in the connection of the perturbation theoretical calculation it was discussed that magnetic flux going through one loop is (since the magnetic field is slowly varying, it can be assumed that the magnetic field B is nearly constant in the region bounded by the loop)

   $$\pi r^2 B = C\frac{v_t^2}{B},$$

   where C is a constant and v_t is the component of the velocity perpendicular to $\mathbf{B}$. Show that

   $$I = \frac{v_t^2}{B}$$

   is an adiabatic invariant i.e. $\dot{I} = 0$.
   Hint: Since there is no electric field present, the kinetic energy of the particle must be conserved, which means that $v^2 = v_l^2 + v_t^2 = \text{Constant}$ (v_l is the component of the velocity parallel to $\mathbf{B}$).

3. Let's assume that the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are time-independent. Show that the scalar and vector potentials can also be chosen time-independent.
   Hint: There always exists time-dependent potentials $\rho(x, y, z, t)$ and $\mathbf{A}(x, y, z, t)$. You can utilize the knowledge about fields ($\partial\mathbf{E}/\partial t = 0$ and $\partial\mathbf{B}/\partial t = 0$) to show, that there exists a suitable function f for gauge transformation such that the new potentials ϕ' and $\mathbf{A}'$ are time-independent.
   You also need to use the familiar result concerning the conservative vector fields: Let's assume that $\mathbf{\Lambda}$ is a vector field. Then $\nabla \times \mathbf{\Lambda} = 0$ if and only if there exists a scalar potential function λ such that $\mathbf{\Lambda} = \nabla\lambda$.