1. Use the definition of the electromagnetic field tensor to show, that

$$(F_{ki}) = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

and

$$(F^{ki}) = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}.$$

2. Let's consider the equation of motion

$$mc\frac{du_k}{ds} = eF_{ki}u^i. \tag{1}$$

Show that the spatial components of the equation 1 lead to equation of motion for 3-vectors

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E} + e\mathbf{v} \times \mathbf{B}$$

obtained earlier.
Show that the time-component of equation 1 gives the relation

$$\frac{dE_{kin}}{dt} = e\mathbf{v} \cdot \mathbf{E}$$

also obtained earlier with different kind of derivation.

3. The results obtained in problem 1 show that the electric and magnetic field components are related to the components of the electromagnetic field tensor $F^{ik}$. So the transformation of these fields in the Lorentz transformation is determined by the transformation of the components of $F^{ik}$ in the Lorentz transformation.
Let's consider a situation, where inertial coordinate frame $K'$ is moving in $x$ direction with velocity $V$ relative to the inertial coordinate frame $K$. Show that

$$B_y = \frac{B'_y - \frac{V}{c^2}E'_z}{\sqrt{1 - \frac{V^2}{c^2}}},$$

where $B_y$ is the magnetic field component in the coordinate frame $K$, and $B'_y$ and $E'_z$ are magnetic and electric field components in coordinate frame $K'$.

4. Show by direct calculation that

$$F^{ik}F_{ik} = -\frac{2}{c^2}(E^2 - c^2B^2)$$

and

$$e^{iklm}F_{ik}F_{lm} = -\frac{8}{c}\mathbf{E}\cdot\mathbf{B}.$$