

1. Use the Lorentz transformation to show that the electric field caused by a point charge moving with constant velocity \mathbf{v} is

$$\mathbf{E}(\mathbf{R}) = \frac{e}{4\pi\epsilon_0} \frac{\mathbf{R}}{R^3} \frac{1 - \frac{v^2}{c^2}}{(1 - \frac{v^2}{c^2} \sin^2 \theta)^{3/2}}.$$

Here \mathbf{R} is the displacement from the charge at the given moment and θ is the angle between \mathbf{v} and \mathbf{R} . Draw a conclusion that the electric field caused by charged particle moving with large velocity can be seen as a short pulse at angle $\theta = \pi/2$ ($\Delta\theta \propto \sqrt{1 - v^2/c^2}$).

2. Use Maxwell's equations in 3-vector form and the generalization of equation

$$\frac{dE_{kin}}{dt} = e\mathbf{v} \cdot \mathbf{E}$$

to derive the law for conservation of energy

$$\frac{d}{dt} \left[\sum E_{kin} + \int dV \left(\frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2 \right) \right] = - \oint d\mathbf{a} \cdot \mathbf{S}.$$

Here \mathbf{S} is the Poynting vector

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}.$$