

1. Calculate

$$\text{a) } \nabla \left(\frac{1}{r^2} \right) \quad \text{b) } \nabla \cdot \left(\nabla \frac{1}{r} \right) \quad \text{c) } \nabla \left(\frac{x-y}{r^n} \right)$$

In which coordinate system each of the calculations is easiest to carry out? Note that the expression in b) vanishes.

2. Calculate the integral

$$\int_{\mathbb{R}^3} dV \frac{e^{-ar}}{r},$$

where dV is a three-dimensional volume element and $a \in \mathbb{R}$ is a constant.

3. Diffraction from a single slit. Derive the intensity distribution

$$I = I_0 \frac{\sin^2 \beta}{\beta^2}$$

of the interference pattern. Here $\beta = \frac{1}{2}kb \sin \theta$, b is the width of the slit and k is the wave number of the radiation field. What happens in the limit $kb \rightarrow 0$? Give a brief physical interpretation for that limit. *You may find the Huygens' principle useful. For a detailed discussion on the single slit diffraction, see e.g. Introduction to Optics, Pedrotti and Pedrotti, second edition.*

4. In the double-slit experiment, there are two slits (width b), whose corresponding points are separated by a distance a . The intensity distribution is

$$I = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \alpha,$$

where β is the same quantity as in problem 3 and $\alpha = \frac{1}{2}ka \sin \theta$. Derive this formula.

For a detailed discussion on the double slit diffraction, see the reference given in problem 3.

5. Solve the differential equation

$$y''(x) - 3y'(x) + 2y(x) = \cos x.$$

See e.g. Fundamentals of Differential Equations and Boundary Value Problems, Nagle, second edition, for a discussion on how to solve differential equations of this form.