1. Show that in the xz-plane the curve
$$r = |Y_1^1(\theta, 0)|$$
represents two circles.

2. Show that the spherical harmonics $Y_m^l(\theta, \varphi)$ can be written in terms of the harmonic polynomials as
   a) $Y_0^0(\theta, \varphi) = \sqrt{\frac{1}{4\pi}}$.

   b) $rY_0^1(\theta, \varphi) = \sqrt{\frac{3}{4\pi}}\, z$,

      $rY_{\pm 1}^1(\theta, \varphi) = \mp\sqrt{\frac{3}{8\pi}}\,(x \pm iy)$.

   c) $r^2Y_0^2(\theta, \varphi) = \sqrt{\frac{5}{16\pi}}\,(2z^2 - x^2 - y^2)$,

      $r^2Y_{\pm 1}^2(\theta, \varphi) = \mp\sqrt{\frac{15}{8\pi}}\, z(x \pm iy)$,

      $r^2Y_{\pm 2}^2(\theta, \varphi) = \sqrt{\frac{15}{32\pi}}\,(x \pm iy)^2$.

3. Construct the $N = 2$ states for an isotropic harmonic oscillator in both Cartesian and spherical coordinates.

4. Write the harmonic oscillator energy eigenfunction $\psi_{2,2,0}(r, \theta, \varphi)$ as a linear combination of the harmonic oscillator energy eigenfunctions $\psi_{n_x n_y n_z}(x, y, z)$.

5. Let us assume that a particle experiences the potential
$$V(x, y, z) = \frac{1}{2}m\left[\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2\right].$$
Determine the energy levels. After that, pick a couple of levels and determine their degeneracies when $\omega_x{=}\omega_y{=}\omega_0$ and $\omega_z{=}\omega_0 + \Delta$ (i.e. when ω_x and ω_y are equal and ω_z differs from them).