1. Solve problem 6 of Exercise 11.

2. Solve problem 6 of Exercise 12.

3. Consider a two-state system whose Hamiltonian operator $\hat{H}$ has eigenvalues E_0 and E_1 and the corresponding orthonormal eigenfunctions $\psi_0(x)$ and $\psi_1(x)$, respectively. Assume that the system is described by a wave function
$$\Psi(x) = A\psi_0(x) + B\psi_1(x),$$
where A and B are assumed real.
(a) What are the possible values for A and B?
(b) What are the possible values for B if E_0 is measured with probability $\frac{1}{4}$?
(c) Let $\hat{M}$ be a Hermitean operator corresponding to a dynamical variable μ. Assume that m is an eigenvalue of $\hat{M}$ with an eigenfunction
$$\Phi(x) = \frac{1}{\sqrt{2}}\psi_0(x) - \frac{1}{\sqrt{2}}\psi_1(x).$$
Show that the probability $P(m)$ of measuring m in state $\Psi(x)$ is
$$P(m) = \frac{1}{2}(1 - 2AB).$$

4. Let us consider a measurement of the momentum of a one-dimensional quantum particle.
(a) Show that the eigenfunctions of the momentum operator are plane waves.
(b) What can you say about the position of the particle right after the momentum measurement? How does this relate with the Heisenberg uncertainty principle? Justify your answer!

5. Let us consider the two-state system of problem 3. Let $\hat{H}_1 = \Delta\hat{\sigma}_x$ be a small perturbation ($\Delta \ll E_1 - E_0$) where $\hat{\sigma}_x$ is an operator that flips an energy eigenstate, i.e.
$$\hat{\sigma}_x\psi_0(x) = \psi_1(x) \quad , \quad \hat{\sigma}_x\psi_1(x) = \psi_0(x).$$
(a) Show that the first order perturbative corrections to the energies E_0 and E_1 vanish.
(b) Show that in the second order, the correction ε to the level separation is
$$\varepsilon = \frac{2\Delta^2}{E_1 - E_0}$$