1. The wave function of a particle in one dimension is
$$\Psi(x,t) = xe^{-2|x|-i\omega t}.$$
a) Calculate the probability density.
b) Let us consider intervals of the form $[x - \frac{1}{4},\, x + \frac{1}{4}]$, where $x \in \mathbb{R}$. Determine the interval in which the particle is most likely found.
c) What is the probability to find the particle at the range $[-2, 2]$?
Hint: Part b) is the most challenging.

2. A particle is described by the wave function
$$\Psi(r) = \frac{e^{(ik-a)r}}{r}; \qquad a > 0 \text{ constant.}$$
Calculate the probability current density $\mathbf{S}$, when $r^2 = x^2 + y^2 + z^2$. How does $\mathbf{S}$ behave for large values of r?

3. A particle is described by the wave function
$$\Psi(\mathbf{r}, t) = \frac{1}{N} e^{-2br} e^{-i\omega t}; \qquad b > 0 \text{ constant.}$$
First, calculate the normalization factor N so that
$$\int |\Psi(\mathbf{r}, t)|^2 \, dV = 1.$$
Then, calculate the expectation values of momentum and energy.
Hint: The expectation value of momentum vanishes.

4. Let us consider a free particle in three dimensions. Show that the divergence of the probability current density vanishes. That is, show that
$$\nabla \cdot \mathbf{S} = 0.$$
What does this result mean?

5. By using the separation of variables, solve the one-dimensional free particle Schrödinger equation. That is, use the separation of variables to solve
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = i\hbar\frac{\partial\psi}{\partial t}.$$
Determine the probability density and the probability current density under the condition that the two arbitrary constants appearing in the solution are real. In what case the probability density does not depend on position? In general, if we know that the probability density is time-independent, what can we say about the probability current density?