1. Let us consider a Gaussian wave packet at a such a moment that the position uncertainty Δx is at its minimum. Show that

$$\Delta x \Delta p = \frac{\hbar}{2}.$$

How do the values of Δx and Δp change when time elapses? Briefly explain the behavior of Δp.

2. The first excited state of an iron atom ^{53}Fe collapses to the ground state by emitting a photon of energy 14.4 keV. The lifetime of the first excited state is 141 ns.
a) Estimate the linewidth ΔE of the spectral line.
b) Calculate the recoil energy of the atom.
Hint: The mass of Iron-53 is given e.g. at http://en.wikipedia.org/wiki/Iron-57.

3. Prove the following commutation relations
a) $[A, B] = -[B, A]$
b) $[A, (B + C)] = [A, B] + [A, C]$
c) $[A, BC] = [A, B]C + B[A, C]$
d) $[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$
e) $[A, B]^\dagger = [B^\dagger, A^\dagger]$
Show that a)-c) hold also for Poisson brackets.

4. Show that the operators corresponding to the Cartesian components of the angular momentum satisfy the commutation relation

$$[L_x, L_y] = i\hbar L_z.$$

Show that the indices x, y, and z in the above equation can be cyclically permuted ($x \to y \to z \to x$). In addition, calculate $\mathbf{L} \times \mathbf{L}$, where $\mathbf{L}$ is the angular momentum operator.

5. Show that the probability $P(b_m, t)$ is a Gaussian function when the weight function $c(E)$ is such. Calculate the mean square deviations of time and energy using these distributions and show that

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Hint: This problem is related to the lecture notes section 8.2.