1. Find the simultaneous eigenfunctions of the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\phi^2}$$

and the angular momentum operator L_z.

2. Sketch the probability distributions of the quantum ring Hamiltonian eigenfunctions

$$\begin{array}{lrcl} \text{(i)} & \psi_3(\phi) & = & \cos 3\phi, \\ \text{(ii)} & \psi_4(\phi) & = & \cos 4\phi. \end{array}$$

Use the orbital representation introduced in the lectures.

3. Construct an explicit expression for the wave packet

$$\psi(\phi) = N \sum_{m_I=0}^{\infty} \frac{1}{m_I!} e^{i m_I \phi}.$$

Determine N so that the wave packet is normalized to unity (employ the result $\int_0^{2\pi} e^{2\cos\phi} d\phi \approx 2\pi \times 2.280$). Sketch the form of $|\psi(\phi)|^2$ for $0 \le \phi \le 2\pi$.

4. Calculate $\langle\phi\rangle$, $\langle\sin\phi\rangle$, and $\langle L_z\rangle$ for the wave packet of the previous problem. You may need the integral $\int_0^{2\pi} \cos\phi e^{2\cos\phi} d\phi \approx 2\pi \times 1.591$.

5. Show that $[\mathbf{L}, r] = 0$ and $[H, \mathbf{L}] = 0$, where $r^2 = x^2 + y^2 + z^2$ and the potential energy part of the Hamiltonian depends only on r (as opposed to $\mathbf{r}$).