Instructions on how to solve exercise 11 problem 6

Apply the lecture notes discussion up to Eq. (132), which reads

$$\nabla^2\psi(\mathbf{r}) + \left(E + \frac{2}{r}\right)\psi(\mathbf{r}) = 0. \tag{1}$$

Note that in this problem, however, the Laplace operator and the position vector are two-dimensional. Write the Laplacian in polar coordinates and employ the separation of variables. You obtain an equation both for the angular and the radial part, namely

$$\begin{aligned} Y''(\varphi) &= -m^2 Y(\varphi), & (2) \\ rR' + r^2R'' + 2rR + Er^2R &= m^2R. & (3) \end{aligned}$$

Solve the angular equation (you obtain a condition for the constant m). After that, substitute $R = u/\sqrt{r}$ into to the radial equation. You obtain

$$u'' + \left(E + \frac{2}{r} - \frac{\left(m - \frac{1}{2}\right)\left(m - \frac{1}{2} + 1\right)}{r^2}\right)u = 0. \tag{4}$$

That is, you obtain the lecture notes Eq. (133) with the difference that l has been replaced by $m - 1/2$. Apply the discussion given in the lecture notes.