EXAMPLE: TWO ELECTRON SYSTEM

- LET US CONSIDER 2 ELECTRONS
- CLEARLY $P_{12}|2e\rangle = -1|2e\rangle$
- BASE KETS ARE SPESIFIED BY

$\bar{x}_1, \bar{x}_2, m_{s1}, m_{s2}$ WITH

$$\psi = \sum_{m_{s1}} \sum_{m_{s2}} c(m_{s1}, m_{s2}) \langle \bar{x}_1 m_{s1}; x_2 m_{s2} | \alpha \rangle$$

- IF THE HAMILTONIAN COMMUTES WITH $S_0^2 = (S_{01} + S_{02})^2$: $[S^2, H] = 0$

THEN THE WAVE FUNCTION SEPARATES

$$\psi = \varphi(\bar{x}_1, \bar{x}_2)\, \chi(m_{s1}, m_{s2})$$

$$\chi(m_{s1}, m_{s2}) \begin{cases} \left.\begin{array}{l} \chi_{++} \\ 1/\sqrt{2}\,(\chi_{+-} + \chi_{-+}) \\ \chi_{--} \end{array}\right\} \text{SYMMETRICAL TRIPLET} \\ \left.1/\sqrt{2}\,(\chi_{+-} - \chi_{-+})\right\} \text{ANTI-SYMM. SINGLET} \end{cases}$$

(1)

THE PERMUTATION OPERATOR INTERCHANGES POSITION AND SPIN, AND CAN BE WRITTEN

$$P_{12} = P_{12}^{SPACE} P_{12}^{SPIN}$$

EXPLICITELY, IN

$$|\alpha\rangle \rightarrow P_{12}|\alpha\rangle$$

$$\varphi(\bar{x}_1, \bar{x}_2) \rightarrow \varphi(\bar{x}_2, \bar{x}_1)$$

$$\chi(m_{s1}, m_{s2}) \rightarrow \chi(m_{s2}, m_{s1})$$

ON THE OTHER HAND,

F-D STATISTICS:

$$\psi(\bar{x}_1 m_{s1}; \bar{x}_2 m_{s2}) = \langle \bar{x}_1 m_{s1}; \bar{x}_2 m_{s2} | \alpha \rangle$$

$$= -\psi(\bar{x}_2 m_{s2}; \bar{x}_1 m_{s1}) = -\langle \bar{x}_2 m_{s2}; \bar{x}_1 m_{s1} | \alpha \rangle$$

WE SEE

i) IF $\varphi(\bar{x}_1, \bar{x}_2)$ SYMMETRIC

$\rightarrow \chi(m_{s1}, m_{s2})$ ANTI-SYMMETRIC

ii) φ ANTI-SYMMETRIC

$\rightarrow \chi$ SYMMETRIC

AS USUAL, THE PROBABILITY TO FIND ELECTRON 1 AROUND $\bar{x}_1$ AND ELECTRON 2 AROUND $\bar{x}_2$ READS

$$|\varphi(\bar{x}_1, \bar{x}_2)|^2 d^3x_1 d^3x_2$$

LET US NOW CONSIDER

2 NON-INTERACTING ELECTRONS

AND LET US IGNORE SPIN (*):

(*) HAMILTONIAN DOES NOT DEPEND ON SPIN

$$\left[-\frac{\hbar^2}{2m}\left(\nabla_1^2+\nabla_2^2\right)+V_{ext}(\bar{x}_1)+V_{ext}(\bar{x}_2)\right]\Psi=E\Psi$$

THE SOLUTION IS OF FORM

$$\omega_A(\bar{x}_1)\omega_B(\bar{x}_2)\chi(m_{s1},m_{s2})$$

DEPENDING ON χ, SPACE PART

IS $\varphi(\bar{x}_1,\bar{x}_2)=\frac{1}{\sqrt{2}}\left(\omega_A(\bar{x}_1)\omega_B(\bar{x}_2)\pm\omega_A(\bar{x}_2)\omega_B(\bar{x}_1)\right)$

$+$: SPIN SINGLET χ

$-$: SPIN TRIPLET χ

$$|\varphi(\bar{x}_1,\bar{x}_2)|^2 d^3x_1 d^3x_2$$

$$= \frac{1}{2}\left[\omega_A^*(\bar{x}_1)\omega_B^*(\bar{x}_2) \pm \omega_A^*(\bar{x}_2)\omega_B^*(\bar{x}_2)\right]$$

$$\times \left[\omega_A(\bar{x}_1)\omega_B(x_2) \pm \omega_A(\bar{x}_2)\omega_B(\bar{x}_2)\right] d^3x_1 d^3x_2$$

$$= \frac{1}{2}\left[|\omega_A(\bar{x}_1)|^2|\omega_B(\bar{x}_2)|^2 + |\omega_A(\bar{x}_2)|^2|\omega_B(\bar{x}_1)|^2\right] d^3x_1 d^3x_2$$

$$\pm \frac{1}{2}\left[2\mathrm{Re}(\omega_A(\bar{x}_1)\omega_B(\bar{x}_2)\omega_A^*(\bar{x}_2)\omega_B^*(\bar{x}_1)\right] d^3x_1 d^3x_2$$

THE LAST TERM IS
<u>EXCHANGE DENSITY</u>

(COMES FROM:

$$|(Ae^{i\varphi})^* + Ae^{i\varphi}| = |A(e^{-i\varphi} + e^{i\varphi})|$$

$$= |A2i\cos\varphi| = 2\mathrm{Re}(Ae^{i\varphi *})$$
)

WE SEE THAT IF THE ELECTRONS ARE IN SPIN-TRIPLET STATE

(-) SIGN:

$|\varphi(\bar{x}_1, \bar{x}_1)|$ VANISHES

$\rightarrow$ ELECTRONS CAN NOT OCCYPY THE SAME "SPOT"

IN SPIN-SINGLET STATE

(+) SIGN:

ENHANCED PROBABILITY TO FIND THEM AT THE SAME POINT DUE TO THE EXCHANGE DENSITY

YOU MIGHT TO BEGIN TO FEEL UNEASY: THERE ARE VIRTUALLY INFINITE AMOUNT OF ELECTRONS IN THIS WORLD.

HOW DO WE TAKE THE EXCHANGE OF THEM INTO ACCOUNT?

SPECIFICALLY, LET THERE BE AN ELECTRON IN MICHAEL JACKSONS NOSTRIL IN CALIFORNIA COUNTY JAIL. WHAT IS THE EFFECT IF YOU EXCHANGE THAT WITH ONE FROM YOUR BRAIN?

THE SOLUTION DEPENDS ON THE WAVE FUNCTIONS.

LET e_1 AT CALIFORNIA

e_2 AT OULU

IF $w_1(\bar{x})$ AND $w_2(\bar{x})$

DO NOT OVERLAP

$\rightarrow$ EXCHANGE DENSITY $\rightarrow 0$

AND

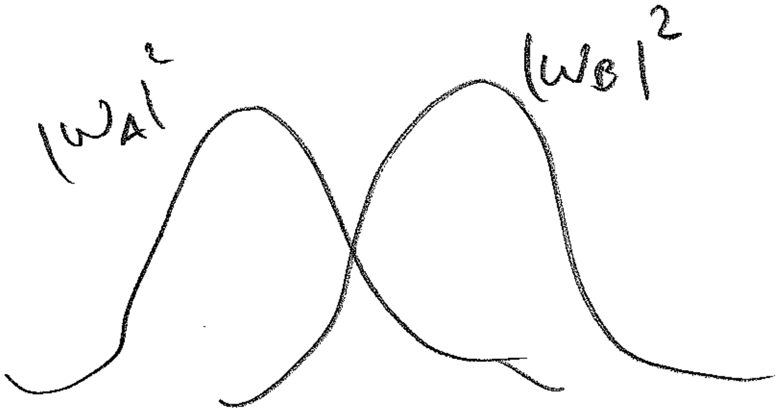
$$|w_A(\bar{x}_1)|^2 |w_B(\bar{x}_2)|^2 + |w_B(\bar{x}_1)|^2 |w_A(\bar{x}_2)|^2$$

ONLY TERM CONTRIBUTES

$$|\varphi|^2 d^3x_1\, d^3x_2 = |w_A(\bar{x}_1)|^2\, |w_B(\bar{x}_2)|^2$$

THIS IS THE CLASSICAL LIMIT (ELECTRONS ARE DEFINITIVELY LOCALIZED)

BUT IN CASE OF OVERLAP



SYMMETRIZATION / ANTISYMMETRIZATION <u>DO</u> PLAY A ROLE