1. Warm up
   a) Show that the eigenvalues of a Hermitian operator A are real and that the eigenkets of A corresponding to different eigenvalues are orthogonal.
   b) Show that if the state ket
   $$|\alpha\rangle = \sum_{a'} c_{a'} |a'\rangle$$
   is normalized then the expansion coeffcients $c_{a'}$ must satisfy
   $$\sum_{a'} |c_{a'}|^2 = 1.$$

2. Prove the Theorem 1 from lecture notes:
   If both of the basis $\{|a'\rangle\}$ and $\{|b'\rangle\}$ are orthonormalized and complete then there exists a unitary operator U so that
   $$|b_1\rangle = U|a_1\rangle, \quad |b_2\rangle = U|a_2\rangle, \quad |b_3\rangle = U|a_3\rangle, \quad \ldots$$
   (Unitary operator: $U^\dagger U = UU^\dagger = 1$)

3. Consider the spin operators S_x, S_y and S_z in the $\{|S_z;\uparrow\rangle, |S_z;\downarrow\rangle\}$ basis
   a) Write out the operators S_x, S_y and S_z in the $\{|S_z;\uparrow\rangle, |S_z;\downarrow\rangle\}$ basis.
   b) Compute the commutators $[S_x, S_y]$ and $[S^2, S_x]$ as well as anticommutator $\{S_x, S_y\}$.
   c) Let us define the ladder operators $S_\pm = S_x \pm iS_y$. Compute $S_\pm |S_z;\uparrow\rangle$ and $S_\pm = |S_z;\downarrow\rangle$.

4. Prove the Theorem 2 from lecture notes:
   If T is a unitary matrix, then the matrices X and $T^\dagger X T$ have the same trace and the same eigenvalues.

5. The translation operator for a finite (spatial) displacement is given by
   $$\mathcal{T}(\mathbf{l}) = \exp\left(-\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{l}\right)$$
   where $\mathbf{p}$ is the momentum operator and $\mathbf{l}$ the displacement vector.
   a) Evaluate $[x_i, \mathcal{T}(\mathbf{l})]$.
   b) How does the expectation value $\langle x\rangle$ of the position operator change under the translation?