1. Consider a three dimensional ket space. If a certain set of orthonormal kets, say $|1\rangle$, $|2\rangle$ and $|3\rangle$ are used as the base kets, then the operators A and B are represented by

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}$$

in which both a and b are real.

   a) Obviously A exhibits a degenerate spectrum. Does B have a degenerate spectrum as well?
   b) Show that A and B commute.
   c) Find a new (orthonormal) set of base kets which are simultaneous eigenkets of both A and B. Specify the eigenvalues of A and B for each of the three eigenkets. Does your specification of eigenvalues completely characterize each eigenket?

2. Evaluate the uncertainty relation of x and p operators for a particle confined in an infinite potential well (between two unpenetrable walls.)

   Some help: In this case the potential can be written:$V(x) = 0$, when $0 < x < a$ and otherwise $V = \infty$. From quantum mechanics we remember that the wave function in such a potential reads

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right),$$

   in which number n refers to the nth excitation while $n = 1$ is the ground state.

3. Show that

$$\langle p' \,|x|\, \alpha\rangle = \mathrm{i}\hbar \frac{\partial}{\partial p'} \langle p'|\alpha\rangle \qquad \text{and} \qquad \langle \beta \,|x|\, \alpha\rangle = \int \mathrm{d}p' \, \phi^\star_\beta(p') \mathrm{i}\hbar \frac{\partial}{\partial p'} \phi_\alpha(p')$$

4. Consider spin precession of electron in static uniform magnetic field in the z direction and calculate the expectation values of spin at time t in y and z directions when the initial state of the system at $t = 0$ is

$$|S_x; \uparrow\rangle = \frac{1}{\sqrt{2}} |S_z; \uparrow\rangle + \frac{1}{\sqrt{2}} |S_z; \downarrow\rangle \,.$$

   Remember the identities:

$$\begin{aligned} |S_y; \uparrow\rangle &= \frac{1}{\sqrt{2}} |S_z; \uparrow\rangle + i\frac{1}{\sqrt{2}} |S_z; \downarrow\rangle \,, \\ |S_y; \downarrow\rangle &= \frac{1}{\sqrt{2}} |S_z; \uparrow\rangle - i\frac{1}{\sqrt{2}} |S_z; \downarrow\rangle \,. \end{aligned}$$