1. **Ensemble of spin $-\frac{1}{2}$ particles, continued**
   Let us have an ensemble of spin-$\frac{1}{2}$ particles. You can think of for example electrons in Stern-Gerlach experiment.
   a) Consider a *pure* ensemble, *i.e.* a case where we have performed and S-G experiment and blocked the other spin component and let the other get through in such a way that we do not actually know which state we have selected. Instead, we know the expectation values of $\langle S_x \rangle$ and $\langle S_z \rangle$ as well as the sign of $\langle S_y \rangle$. How can we find the state vector corresponding to our ensemble?
   b) Consider a *mixed* ensemble of spin-$\frac{1}{2}$ particles in which we do not know the relative fractions of spin up and down states in our ensemble but the ensemble averages $[S_x]$, $[S_y]$ and $[S_z]$ we do know. Find a 2×2 density matrix characterizing our ensemble.

2. **Pauli's spin matricies**
   Show that
$$\exp\left(\frac{-i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}\phi}{2}\right) = \mathbf{1}\cos\frac{\phi}{2} - i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}\sin\frac{\phi}{2}.$$

3. **Coupling of two spin-$\frac{1}{2}$ particles**
   Consider a system of two spin-$\frac{1}{2}$ particles. Derive the Clebsch–Gordan coefficients, *i.e.* the transformation matrix elements from the uncoupled to the coupled base kets, in the case the orbital angular momentum is suppressed. Otherwise stated: couple the spin angular momentum of two spin-$\frac{1}{2}$ particles.

4. **Rotation of an orbital angular momentum state**
   Consider an orbital angular momentum state $|l = 2, m = 0\rangle$. Suppose this state is rotated by an angle β around the$y$ axis. Find the probability for the new state to be found in states $m = -2, -1, 0, 1, 2$.