Quantum Field Theory Final exam, 15 May 2008

Answering correctly to 4 questions gives full points; however, answering to all 5 contributes to the total point score.

In all questions below we shall consider the "scalar quantum electrodynamics", a theory with a complex scalar field $\phi(x)$ and an abelian gauge field $A_\mu(x)$. The action is

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \left(D_\mu(x)\phi(x)\right)^* D^\mu(x)\phi(x) - m^2 \phi^*(x)\phi(x) \right], \tag{1}$$

where

$$D_\mu(x) = \partial_\mu - ieA_\mu(x), \qquad F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x),$$

and e is the coupling constant.

1. Show that the action is invariant under the gauge transformation ($\theta(x)$ a real number)

$$\phi(x) \to e^{i\theta(x)}\phi(x), \;\; \phi^*(x) \to e^{-i\theta(x)}\phi^*(x), \;\;\; A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu \theta(x).$$

2. Use the Euler-Lagrange equations (here written for the scalar field)

$$\frac{\delta \mathcal{L}}{\delta \phi^*(x)} - \partial_\nu \frac{\delta \mathcal{L}}{\delta(\partial_\nu \phi^*(x))} = 0.$$

to derive the classical equations of motion for both the scalar and the gauge field.

3. a) Write down the Feynman propagator $G(p)$ for the scalar field ϕ.

   b) For the free gauge field, show that we can write

$$\int d^4x - \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) = \int d^4x\, d^4y \frac{1}{2} A_\mu(x) [g^{\mu\nu}\partial_\alpha \partial^\alpha - \partial^\mu \partial^\nu] \delta^4(x-y) A_\nu(y).$$

   c) Using the Lorenz gauge $\partial_\nu A^\nu = 0$, argue that the gauge field Feynman propagator is

$$D_{\mu\nu}(q) = \frac{-ig_{\mu\nu}}{q^2 + i\epsilon}.$$

   (Note that in this case you need not to use the gauge parameter ξ, but can impose the gauge condition directly. Ignore ghosts.)

4. Going back to the full action (1), identify the two interaction terms. Draw the corresponding vertices.

5. Draw all connected Feynman diagrams for the 2-, 3- and 4-point functions for both scalar and gauge field, up to and including order e^2. What are the corresponding symmetry factors?