1. **RLC-circuits**
Derive the electric oscillator equations (8) and (9) of the lecture notes.

2. **Damped harmonic oscillator: exact solution**
The equation of motion for a damped harmonic oscillator under external force can be reduced to (a set of equations of) the form

$$\ddot{q} + \gamma\dot{q} + \omega_0^2 q = A\cos(\omega_d t),$$

where A is a constant. Find the general solution of the equation.

Hint: Treat q as complex and find one solution of the (inhomogeneous) equation of the form $q = Be^{i\omega_d t} + Ce^{-i\omega_d t}$, where B and C are constants. In addition you need to find the general solution of the corresponding homogeneous equation $\ddot{q} + \gamma\dot{q} + \omega_0^2 q = 0$.

3. **Damped harmonic oscillator: RWA solution**
The solution of the previous problem in the rotating wave approximation is

$$\alpha(t) = \frac{f_0}{\omega_0 - \omega_d - \frac{i}{2}\gamma}(e^{-i\omega_d t} - e^{-i\omega_0 t - \frac{\gamma}{2}t}) + \alpha(0)e^{-i\omega_0 t - \frac{\gamma}{2}t}.$$

Plot the real and imaginary part of $\alpha(t)$ as a function of time, and depict the trajectory of $\alpha(t)$ in the complex plane for
(a)

$$f_0 = 0,\ \alpha(0) = 1,\ \omega_0 = 1,\ \gamma = 0.1;$$

(b)

$$f_0 = 1,\ \alpha(0) = 0,\ \omega_0 = \omega_d = 1,\ \gamma = 0.1,$$

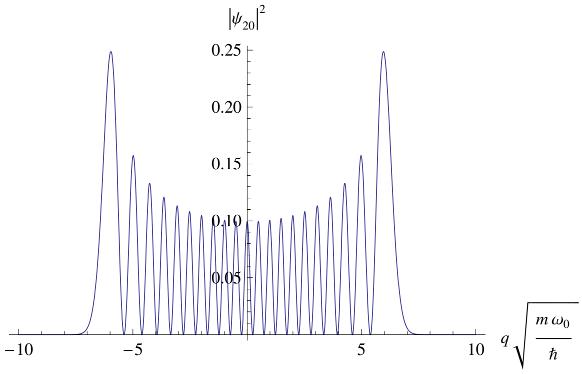
and interpret the results.

4. **RWA steady state properties**
Plot the real and imaginary parts (proportional to the dispersion and absorption) and the phase δ of the steady state solution

$$e^{i\omega_d t}\alpha_{ss}(t) = \alpha_{ss}(0) = \frac{f_0}{\omega_0 - \omega_d - \frac{i}{2}\gamma} = Ae^{i\delta},$$

as a function of ω_d, and depict its trajectory in the complex plane for $f_0 = 1$, $\omega_0 = 1$, $\gamma = 0.1$ and interpret the results.

1. **Comparison between the quantum and classical distributions**
   A particle, in a harmonic potential, has total energy $20.5\hbar\omega_0$. Find the classical probability distribution to find it within $[q, q+\mathrm{d}q]$. Compare the classical probability distribution (qualitatively) with the quantum mechanical probability distribution $|\psi_{20}(q)|^2$ shown in the figure.



2. **Analytical solution for coupled harmonic oscillators**
   The Lagrange function describing coupling of the oscillator 0 to N other oscillators is

$$L = \frac{1}{2}m_0\dot{q}_0^2 - \frac{1}{2}m_0\omega_0^2 q_0^2 + \sum_{i=1}^{N}\left(\frac{1}{2}m_i\dot{q}_i^2 - \frac{1}{2}m_i\omega_i^2 q_i^2 + q_0 g_i q_i\right).$$

   According to classical mechanics, find the eigenfrequencies and eigenvectors in the case of $N = 1$. For simplicity assume $\omega_1 = \omega_0$. Write the solution for the initial values $q_0(0) = 1$, $q_1(0) = \dot{q}_0(0) = \dot{q}_1(0) = 0$.

3. **Hamilton's equations of motion**
   Derive the Hamilton's equations of motion corresponding to the Hamiltonian

$$H = \frac{p_0^2}{2m_0} + \frac{1}{2}m_0\omega_0^2 q_0^2 + \sum_{i=1}^{N}\left(\frac{p_i^2}{2m_i} + \frac{1}{2}m_i\omega_i^2 q_i^2 - q_0 g_i q_i\right),$$

   where the index $i = 0$ refers to the primary oscillator.

4. **Coupling to the environment: Numerical simulation**
   Solve the preceding equations of motion numerically using Mathematica and the "NDSolve" command for an environment consisting of three oscillators. Use the oscillator parameters $m_0 = k_0 = 1$, $k_i = 20$, $m_1 = 19$, $m_2 = 20$ and $m_3 = 21$. For the coupling constants choose $g_i = 0.1$ and start the modeling from the initial values $q_i(0) = p_i(0) = 0$, $q_0(0) = 1$, $p_0(0) = 0$. Plot the solution for $q_0(t)$ and interpret the results. What might be the effect of increasing the number of coupled oscillators?

1. **Vector space and inner product**
   (a) Check that the states
   $$\sum_{n=0}^{\infty} \lambda_n |n\rangle,$$
   satisfy the requirements set for a vector space as stated in the two first sets of equations in Sectinon 3.1 of the lecture notes. The states $|n\rangle$ are Fock states (i.e. eigenstates of simple harmonic oscillator), and λ_n are complex valued.
   (b) Show that the inner product defined by
   $$\langle m|n\rangle = \delta_{m,n}.$$
   satisfies the requirments of the inner product. The states $|n\rangle$ belong to the above vector space and the states $\langle m|$ in the corresponding dual space.
   (c) Check that $a^\dagger$ is a Hermitian conjugate of a and that $a^\dagger a$ is a Hermitian operator.

2. **Schrödinger, Heisenberg and interaction pictures**
   (a) Making a change of coordinates $A' = U^\dagger A U$ (from the Schrödinger picture) show that the equations of motion (lecture notes) transform to
   $$i\hbar \frac{d|a'\rangle}{dt} = \left(U^\dagger H U + i\hbar \frac{dU^\dagger}{dt} U \right) |a'\rangle$$
   $$i\hbar \frac{dA'}{dt} = \left[A', -i\hbar \frac{dU^\dagger}{dt} U \right] + i\hbar U^\dagger \frac{\partial A}{\partial t} U.$$
   In other words, calculate the intermediate steps from Eq. (79) to (81).
   (b) Further, derive the corresponding equations in the Heisenberg and in the interaction picture. In the Heisenberg picture, the transformation U is defined as
   $$i\hbar \frac{dU}{dt} = HU,$$
   and in the interaction picture
   $$i\hbar \frac{dU}{dt} = H_0 U,$$
   where H_0 is time-independent.

3. **Interaction picture: time evolution of operators**
   Find the operators a and $a^\dagger$ in the interaction picture choosing H_0 as Hamiltonian of the simple harmonic oscillator.

4. **Density matrix: a two-state system**
   Calculate the density matrix and the expectation value of $S_x = \sigma_x/2$ in case of
   (a) equal mixture of $|\uparrow\rangle$ and $|\downarrow\rangle$
   (b) pure state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$.
   The states $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of the operator $S_z = \sigma_z/2$.

5. **Density matrix: properties**
   Show that $\mathrm{Tr}\rho = 1$ (note that states $|a_i\rangle$ are normalized but not necessarily orthogonal).

6. **Density matrix: equation of motion**
   Using the Schrödinger equation verify the equation of motion for the density matrix in the Schrödinger picture
   $$i\hbar\frac{d\rho}{dt} = [H, \rho],$$
   and the same in the interaction picture
   $$i\hbar\frac{d\rho'}{dt} = [H_1', \rho'].$$

1. **Thermal distribution**
   Starting from the Gibbs distribution, derive the two forms of the density operator for a simple harmonic oscillator as given in the lectures.

2. **Commutator expressed as a matrix**
   Consider the commutator
   $$B = [A, \rho],$$
   where A and ρ are 2×2 matrices. Express the commutator as a 4×4 matrix U by considering ρ and B as 4-dimensional vectors, so that
   $$B = U\rho.$$

3. **Master equation**
   On the equation
   $$\frac{d\rho}{dt} = -\int_0^t dt_1 \mathrm{Tr}_R[(a^\dagger\Gamma(t)e^{i\omega_0 t} + a\Gamma^\dagger(t)e^{-i\omega_0 t}),$$
   $$[(a^\dagger\Gamma(t_1)e^{i\omega_0 t_1} + a\Gamma^\dagger(t_1)e^{-i\omega_0 t_1}), \rho_R(0)\rho(t)]],$$
   select freely another term (besides the one presented in the lectures) that will lead to a nonvanishing contribution in the master equation and write what are the differences in the subsequent calculation.

4. **Cauchy's formula**
   Justify the relation
   $$\lim_{\epsilon\to 0}\int_{-\infty}^{\infty} dx \frac{f(x)}{x + \mathrm{i}\epsilon} = -\mathrm{i}\pi \int_{-\infty}^{\infty} dx f(x)\delta(x) + P\int_{-\infty}^{\infty} dx \frac{f(x)}{x},$$
   where P denotes principal value integration at $x = 0$.

   Hint: Consider separately the real and imaginary parts of $\frac{1}{x+\mathrm{i}\epsilon}$.

5. **Damped harmonic oscillator**
   Verify the equations
   $$\frac{d\langle a\rangle}{dt} = -i\omega_0\langle a\rangle - \frac{\gamma}{2}\langle a\rangle + if_0 e^{-i\omega_d t},$$
   and
   $$\frac{d\langle a^\dagger a\rangle}{dt} = if_0(\langle a^\dagger\rangle e^{-i\omega_d t} - \langle a\rangle e^{i\omega_d t}) - \gamma\langle a^\dagger a\rangle + \gamma N.$$

1. **Correlation functions in thermal state**
   Let us consider a simple harmonic oscillator (neither driving nor damping) in equilibrium at temperature T. Calculate $\langle a(t)\rangle$, $\langle a^\dagger(t)\rangle$, and the correlation functions $\langle a(t)a^\dagger(0)\rangle$ and $\langle a^\dagger(t)a(0)\rangle$. Try to interpret the results for $T = 0$ and $T > 0$.

2. **Detailed balance**
   Verify that the Bose distribution

$$\rho = \sum_{n=0}^{\infty} |n\rangle\langle n| \frac{1}{1+N}\left(\frac{N}{1+N}\right)^n, \qquad N = \frac{1}{\exp(\beta\hbar\omega_0) - 1},$$

   satisfies the detailed balance condition

$$t_+(n)P(n) = t_-(n+1)P(n+1),$$

   where

$$t_+(n) = \gamma N(n+1), \qquad t_-(n) = \gamma(N+1)n, \qquad P(n) \equiv \rho_{nn}.$$

3. **Temperature relaxation**
   Show that the time dependent Bose distribution ($t \geq 0$) whose diagonal elements are defined as

$$\rho_{nn} = \frac{1}{1+N'}\left(\frac{N'}{1+N'}\right)^n, \qquad N' = N(1 - e^{-\gamma t}),$$

   satisfies the master equation

$$\frac{\mathrm{d}P(n)}{\mathrm{d}t} = t_+(n-1)P(n-1) + t_-(n+1)P(n+1) - [t_+(n) + t_-(n)]P(n),$$

   where transition rates $t_\pm(n)$ and the occupation probability $P(n)$ are exactly as above. Find the momentary temperature of the distribution.

4. **Glauber states**
   The Glauber states $|\alpha\rangle$ are represented with number states as

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle,$$

   where α is an arbitrary complex number. Show the following properties of the Glauber states:

(a) The mean number and the standard deviation of the photon number are

$$\langle \hat{n} \rangle = |\alpha|^2, \qquad \Delta n = \sqrt{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2} = \sqrt{\langle \hat{n} \rangle},$$

respectively.

(b) The distribution of photon numbers follows the Poissonian distribution i. e.

$$P(n) = |\langle n|\alpha\rangle|^2 = e^{-\langle \hat{n} \rangle} \frac{\langle \hat{n} \rangle^n}{n!}.$$

(c) The overlap of two Glauber states $|\alpha\rangle$ and $|\beta\rangle$ depends exponentially on the mutual distance of the coordinates α and β on the complex plane:

$$|\langle \alpha|\beta\rangle|^2 = \mathrm{e}^{-|\alpha-\beta|^2}.$$

1. **Cat states**
   Verify that $\rho = ||\alpha\rangle\langle\beta|| e^{-\alpha\beta^*}$ is a solution of the master equation

$$\frac{d\rho}{dt} = \frac{\gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a),$$

   if $\alpha(t)$ and $\beta(t)$ satisfy the corresponding classical equations. The unnnormalized Glauber states are defined by

$$||\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \,.$$

2. **Number states represented as pointer states**
   Verify first the completeness relation of the Glauber states

$$\frac{1}{\pi} \int |\alpha\rangle \langle\alpha| \; \mathrm{d}^2\alpha \; = 1$$

   and after that find the representation of a number state $|n\rangle$ as a superposition of Glauber states

$$|n\rangle = \int d^2\alpha f(\alpha)|\alpha\rangle.$$

   Hint: The integral $\int |\alpha\rangle \langle\alpha| \; \mathrm{d}^2\alpha$ is over the whole complex plane. Split the integral in the radial and angular parts. Use Mathematica to solve the radial integration, or use integration by parts repeatedly.

3. **Anharmonic oscillator**
   Consider a particle in a sinusoidal potential described by the Hamiltonian

$$H = \frac{p^2}{2m} - m\omega_0^2 q_0^2 \cos\left(\frac{q}{q_0}\right),$$

   where $q_0 = \sqrt{\hbar/m\omega_0}$. If the particle is trapped nearby the potential energy minimum $q = 0$, it can be described as a harmonic oscillator using the expansion $\cos(q/q_0) \approx 1 - \frac{1}{2!}(q/q_0)^2$.

   Find the corrections to the oscillator's energy levels due to the first anharmonic term $\frac{1}{4!}(q/q_0)^4$ of the expansion. Use the first order perturbation theory and express q^4 in terms of a and $a^\dagger$.

1. **Cooper-pair-box**
   By treating the Josephson junction as a capacitor C_J in parallel with a Josephson potential energy $-E_J \cos\left(\frac{2\pi\Phi}{\Phi_0}\right)$, show that the Hamiltonian of the Cooper-pair-box can be presented in the form

$$H_{\text{CPB}}(\Phi, Q) = \frac{(Q+Q_0)^2}{2C_\Sigma} - E_J \cos\left(\frac{2\pi\Phi}{\Phi_0}\right),$$

   where $C_\Sigma = C_J + C_g$ and $Q_0 = C_g U$. The circuit diagram is shown after the Eq. (192) in the lecture notes. Note that there is one more closed loop because of the parallel capacitor

2. **Charge qubit**
   The eigenstates of the Cooper-pair-box for $E_J = 0$, labeled as the charge states $|n\rangle$ ($n \in \mathbb{Z}$), correspond to excess number of Cooper pairs on the island between the capacitors. These can be used as a convenient basis for H_{CPB}. The Hamiltonian is then

$$H_{\text{CPB}} = \sum_n \left[4E_c\left(n + \frac{Q_0}{2e}\right)^2 |n\rangle\langle n| - \frac{E_J}{2}(|n+1\rangle\langle n| + |n-1\rangle\langle n|)\right].$$

   It is a sum of a charging energy of the island and a Cooper pair tunneling operator due to the Josephson effect. Assuming that $Q_0 \approx -e$ and $E_J \ll E_c$, find the ground state and the first excited state and their eigenenergies nearby the avoided level crossing point $Q_0 = -e$.

   Hint: Write $Q_0 = (p-1)e$, where p is a small parameter. Only the two lowest charge states $|0\rangle$ and $|1\rangle$ are needed for writing the approximate 2×2 Hamiltonian for this case.

3. **Rabi oscillations**
   Do all the intermediate steps not shown in the lecture notes of the Rabi oscillations, i.e. equations (211)-(218).

1. **Bloch sphere description**
   Verify that the density matrix equation of motion for a two-state system can be written as
   $$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} = \boldsymbol{\Omega} \times \boldsymbol{P}$$
   where the vectors $\boldsymbol{P}$ and $\boldsymbol{\Omega}$ are determined from the relations
   $$\rho = \frac{1}{2}(I + \boldsymbol{P} \cdot \boldsymbol{\sigma}) \qquad\qquad H = cI + \frac{\hbar}{2}\boldsymbol{\Omega} \cdot \boldsymbol{\sigma}.$$

2. **Polarization vector**
   (a) Find the states that correspond to the cases $\boldsymbol{P} = \boldsymbol{i}$, $\boldsymbol{P} = \boldsymbol{j}$ and $\boldsymbol{P} = \boldsymbol{k}$.
   (b) Show that
   $$\boldsymbol{P}^2 \leq 1,$$
   and that the equality holds only for a pure state.

3. **Master equation for the two-state system**
   Verify that the master equation for the two-state system with no driving is explicitly
   $$\begin{aligned}
   \dot{\rho}_{aa} &= -\gamma(N+1)\rho_{aa} + \gamma N \rho_{bb} \\
   \dot{\rho}_{ab} &= -\mathrm{i}\Omega_0 \rho_{ab} - \gamma(N + \frac{1}{2})\rho_{ab} \\
   \dot{\rho}_{ba} &= \mathrm{i}\Omega_0 \rho_{ba} - \gamma(N + \frac{1}{2})\rho_{ba} \\
   \dot{\rho}_{bb} &= \gamma(N+1)\rho_{aa} - \gamma N \rho_{bb}.
   \end{aligned}$$

4. **State manipulation of the two-state system: $\pi/2$-pulse**
   Let us consider an undriven two-state system in a rotating frame: $U = \exp(-\mathrm{i}\Omega_0 t \sigma_z/2)$. Suppose that the two-state system is in the ground state $\boldsymbol{P} = -\boldsymbol{k}$ at the initial time $t = 0$.
   (a) In that frame a gate, i. e. perturbation $H_1 = \frac{\hbar}{2}\Omega_x \sigma_x$, is then switched on at $t = 0$. The coefficient Ω_x is taken as a fixed constant. Find the trajectory $\boldsymbol{P}(t)$ for $t \geq 0$.
   (b) The gate is switched off at $t = T$. Find T which takes the system to a state for which $P_z = 0$ (for $t \geq T$).

1. **Energy balance in a two-level system**
In the case of harmonic oscillator we derived

$$\dot{E} = P_{\text{abs}} - P_{\text{dis}}.$$

Find the corresponding relation for a two-level system using $E = \frac{1}{2}\hbar\Omega_0 P_z$ and the Allen&Eberly Bloch equations. What are the expressions for P_{abs} and P_{dis} in the steady state? Verify that they have a Lorentzian form and calculate their line widths.

2. **Autocorrelations and the spectral density**
Consider a closed system described by the Hamiltonian H at thermal equilibrium. Dynamical properties of a physical quantity A can be studied via the autocorrelation function $R_A(\tau) = \langle A(t)A(t-\tau)\rangle$, where A is the corresponding Hermitian operator.
   (a) Show first that the autocorrelation function is invariant in time translations, i.e.

$$R_A(\tau) = \langle A(\tau)A(0)\rangle .$$

   (b) Show then that it satisfies

$$R_A(-\tau) = (R_A(\tau))^*.$$

   (c) Finally, show that its spectral density

$$S_A(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} R_A(\tau)$$

   is a real valued function and verify that for "classical" correlations, for which $R_A(\tau) = R_A(\tau)^*$, the spectral density is an even function of ω i. e. $S_A(-\omega)=S_A(\omega)$.

3. **Spontaneous vs. induced emission**
Express

$$S_F(\omega) = \frac{2\hbar\omega m\gamma(|\omega|)}{1 - e^{-\frac{\hbar\omega}{k_B T}}}$$

as a sum of two terms that correspond to the induced emission or absorption (vanishes at $T = 0$) and to the spontaneous emission (independent on T). What special symmetry does the induced part have? At what frequency the spontaneous emission becomes the dominant cause of emission at
   (a) room temperature
   (b) $T = 1$ K?

1. **Jaynes-Cummings model** (double points)
   Do the intermediate steps from Eq. (247) to Eq. (250) and find the energy eigenstates $|\pm, n\rangle$ of the JC Hamiltonian

$$H_{\mathrm{JC}} = \frac{\hbar\Omega_0}{2}\sigma_z + \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right) - \hbar g(\sigma^+ a + \sigma^- a^\dagger).$$

   Show that the eigenstates can be expressed in the form

$$|-, n\rangle = \begin{pmatrix} \sin\frac{\theta_n}{2}\,|n\rangle \\ \cos\frac{\theta_n}{2}\,|n+1\rangle \end{pmatrix} \qquad |+, n\rangle = \begin{pmatrix} \cos\frac{\theta_n}{2}\,|n\rangle \\ -\sin\frac{\theta_n}{2}\,|n+1\rangle \end{pmatrix}.$$

   where $\tan\theta_n = \frac{2g\sqrt{n+1}}{\Omega_0-\omega_0}$, $0 \le \theta_n \le \pi$. The above notation means that

$$\begin{pmatrix} \psi_1|n\rangle \\ \psi_2|n+1\rangle \end{pmatrix} = \psi_1\,|a, n\rangle + \psi_2\,|b, n+1\rangle\,, \quad \psi_{1,2} \in \mathbb{C}.$$

2. **Jaynes-Cummings model with dissipation** (double points)
   By coupling the oscillator of the JC model to a bath at $T = 0$ (producing spontaneous emission) we obtain the master equation

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H_{\mathrm{JC}}, \rho] + \frac{\gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a).$$

   This can be represented either in the bare basis $|a/b, n\rangle$ or the dressed basis $|\pm, n\rangle$ of the JC model.

   By cutting the master equation to two lowest states in the dressed basis, write down the resulting equations for the evolution of the density matrix. Plot the decay rate of the excited state as a function of the qubit's level splitting and explain physically what happens for the decay rate.

   Hint: The cut density matrix in the dressed basis is

$$\rho = \rho_{bb}\,|b, 0\rangle\,\langle b, 0| + \rho_{b-}\,|b, 0\rangle\,\langle -, 0| + \rho_{-b}\,|-, 0\rangle\,\langle b, 0| + \rho_{--}\,|-, 0\rangle\,\langle -, 0|\,,$$

   where ρ_{ij} are time-dependent coefficients, $|b, 0\rangle$ is the singlet state and the state $|-, 0\rangle$ is derived in the Ex. 10.1.

1. **Shannon wavelet**
   Calculate the integral
   $$\phi(t) = \int_{\omega_1}^{\omega_2} d\omega e^{-i\omega t}.$$
   Plot this "Shannon wavelet" (neglect the complex phase factor) and the next orthogonal to this obtained by a translation in time.

2. **Transverse modes and the quantum resistance**
   In the lectures the transverse modes were assumed to be a result of a hard wall confinement in the conducting wire. Suppose that the confinement is parabolic. Sketch qualitatively the differences that this would make to the behavior of the conductivity. Also consider how would the "quantum resistance" change if there would be no spin degree of freedom, and if (in addition to this) the smallest charge would be a Cooper pair (treating it as a fermion).

3. **Poisson distribution**
   Consider particles that arrive randomly and independently of each other with a constant rate λ. Justify that the probability $P_n(t)$ for the arrival of n particles during time t obeys the differential equation ($n \geq 1$)
   $$\frac{dP_n}{dt} = \lambda(P_{n-1} - P_n).$$
   For $P_0(t)$ we take the initial condition $P_0(0) = 1$. What is the equation for $P_0(t)$? Show that the solution with initial conditions $P_n(0) = \delta_{n,0}$ is the Poisson distribution
   $$P_n(t) = e^{-\lambda t}\frac{(\lambda t)^n}{n!},$$
   with average $\langle n(t)\rangle = \lambda t$.

4. **Shot noise due to a tunnel junction** (double points)
   Calculate the classical spectral density of current fluctuations across the resistor ($T = 0$) in a voltage-biased ($V = e/C$) RC-circuit (voltage source, resistor and capacitor in series) whose capacitor "leaks" electrons so that they have a constant tunneling rate λ across the capacitor (to the direction that produces positively signed current). Assume that the time between the tunnelings is much larger than that of recharging of the capacitor so that you can treat the tunneling events independent of each other.
   Hint:
   - Calculate the current of recharging the capacitor after leaking an electron.

- Express then the current as a sum of independent reloading events and calculate the mean current $\langle I(t) \rangle$
- Deduce then a non-zero realization of the entity $I(t)I(0)$. Finally calculate the correlator $\langle I(t)I(0) \rangle$ as a sum of the all possible non-zero realizations.